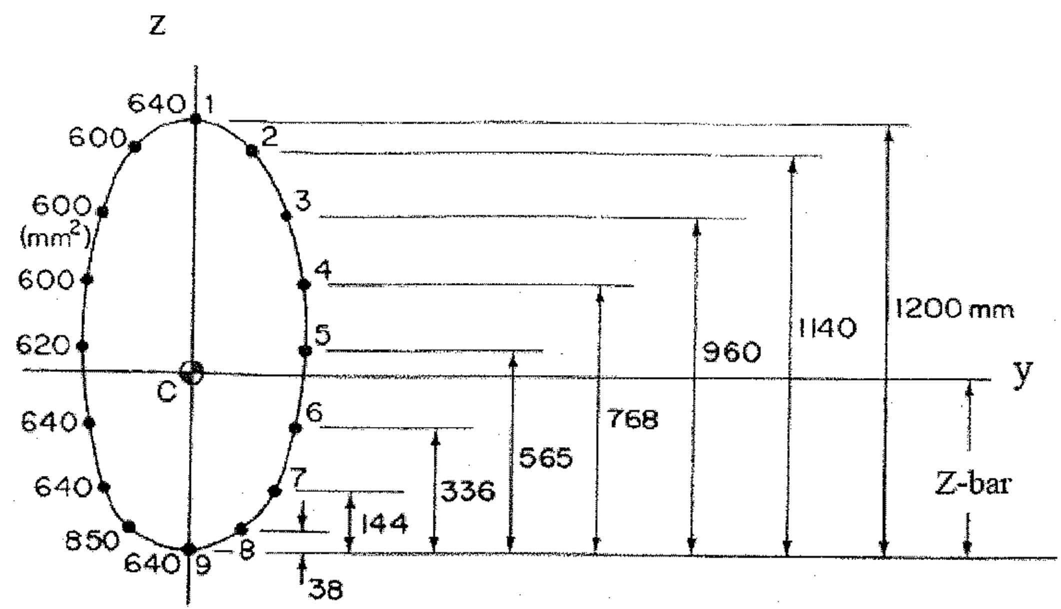
MAE3315
HW # 6. (SOLUTION)

Problem 6.1

The idealized fuselage section as shown in the figure is subjected to a bending moment, M_y of 100 kN-m. The entire fuselage is symmetric with respect to z-axis. The stringer is represented by a solid dots and the fuselage skin is represented a line between dots. The cross-section of the stringer is shown in the figures. The position of each stringer is also shown. Determine the bending stress of each stringer.



Solution:

(Area) x (Number of Stringers)

$\bar{z}_c = \frac{\sum A_i \bar{z}_i}{\sum A_i}$

z-position with respect to the centroid

$I_{yy} = \sum_i A_i z_i^2$

Stresses on each stringer $\sigma = \frac{M_y z_i}{I_{yy}}$

#	z_{ori}-position (mm)	Number of Stringers	Area (mm^2)	Total Area (mm^2)	A_i*z_{ori}/A_{total}	z-position (mm)	I_{yy} (mm^4)	σ (Pa)
1	1200	1	640	640	73.9884	661.5414	280087717	35,669,939
2	1140	2	600	1200	131.7919	601.5414	434222504	32,434,773
3	960	2	600	1200	110.9827	421.5414	213236608	22,729,275
4	768	2	600	1200	88.7861	229.5414	63227119	12,376,744
5	565	2	620	1240	67.4952	26.5414	873515	1,431,099
6	336	2	640	1280	41.4335	-202.4586	52466527	-10,916,452
7	144	2	640	1280	17.7572	-394.4586	199164885	-21,268,983
8	38	2	850	1700	6.2235	-500.4586	425779934	-26,984,443
9	0	1	640	640	0.0000	-538.4586	185560087	-29,033,381

A_{total} = 10380 Z_{bar} = 538.4586

I_{yy} = 1854618897 mm^4

I_{yy} = 0.00185462 m^4

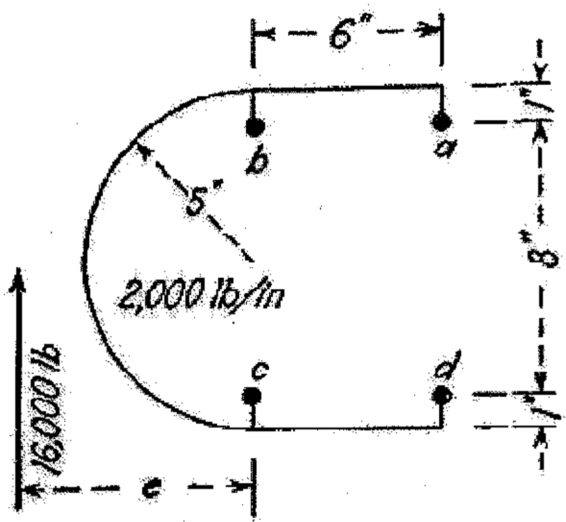
Summation of all the Total Areas

This is $\bar{z}$

Total I_{yy}

Problem 6.2

Find the shear flows in the webs of the beam shown in the figure. Each of the four flanges has an area of 1.0 in^2. Find the shear center for the area.



Solution:

Let's assume the lower-left stringer (c) is the origin,

$$\bar{y}_c = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(6in)(1in^2)+(6in)(1in^2)}{1in^2+1in^2+1in^2+1in^2} = 3in$$

$$\bar{z}_c = \frac{\sum \bar{z}_i A_i}{\sum A_i} = \frac{(8in)(1in^2)+(8in)(1in^2)}{1in^2+1in^2+1in^2+1in^2} = 4in$$

The moment of inertia about the horizontal centroidal axis is,

$$I_y = \sum_i A_i z_i^2 = (1in^2)(4in)^2 + (1in^2)(4in)^2 + (1in^2)(-4in)^2 + (1in^2)(-4in)^2 = 64in^4$$

The shear flows are obtained by,

$$q_{ab} = -\frac{V_z}{I_y} Q_{ab} = -\frac{16{,}000lb}{64in^4}(1in^2)(4in) = -1{,}000lb/in$$

NOTE: The positive shear flow is clockwise

$$q_{bc} = -\frac{16{,}000lb}{64in^4}\left[(1in^2)(4in)+(1in^2)(4in)\right] = -2{,}000lb/in$$

Or q_{bc} it can also be calculated as follows,

$$q_{bc} = q_{ab} - \frac{V_z}{I_y} Q_{bc} = -1{,}000 lb/in - \frac{16{,}000 lb}{64 in^4}\left[\left(1 in^2\right)\left(4 in\right)\right] = -2{,}000 lb/in$$

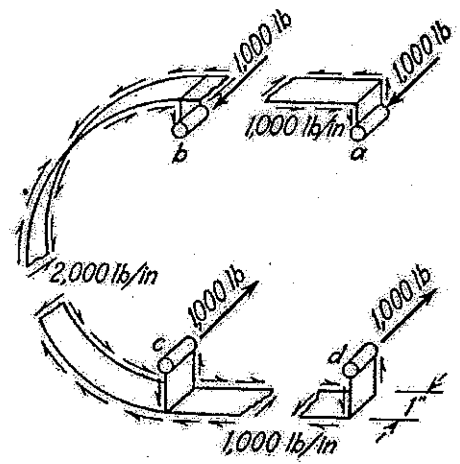
To calculate q_{cd},

$$q_{cd} = -\frac{16{,}000 lb}{64 in^4}\left[\left(1 in^2\right)\left(4 in\right) + \left(1 in^2\right)\left(4 in\right) + \left(1 in^2\right)\left(-4 in\right)\right] = -1{,}000 lb/in$$

Or,

$$q_{cd} = q_{bc} - \frac{V_z}{I_y} Q_{cd} = -2{,}000 lb/in - \frac{16{,}000 lb}{64 in^4}\left[\left(1 in^2\right)\left(-4 in\right)\right] = -1{,}000 lb/in$$

The axial loads and shear flows are shown in the figure.



The distance "e" to the shear center is found by taking moments about a point below c, on the juncture of the webs. The shear flow in the nose skin produces a moment equal to the product of the shear flow and twice the area enclosed by the semicircle. The shear flow in the upper horizontal web has a resultant force of 6,000 lb and a moment arm of 10 in. The short vertical webs at "a" and "d" each resist forces of 1,000 lb, with a moment arm of 6 in. The resultant forces on the other webs pass through the center of moments. Equating the moment of the 16,000 lb external shearing force to the moment of the shear flows,

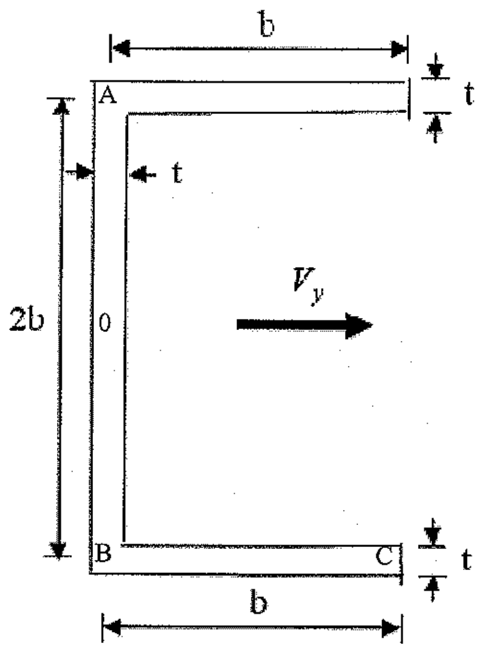
$$16{,}000(e) = (2)\left(\frac{\pi 5^2}{2}\right)(2{,}000) + (6{,}000)(10) + 2(1{,}000)(6)$$

$$\boxed{e = 14.32 in}$$

Problem 6.3

The beam is subjected to **horizontal load, V_y pointing to the right.**

Find shear flow of each segment in the solid C-section (Fig. a) shown below.



Solution:

First, the location of the centroid is calculated,

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{(b/2)(bt) + (0)(2bt) + (b/2)(bt)}{bt + 2bt + bt} = \frac{b}{4}$$

$$z_c = 0$$

The moment of inertia is,

$$I_z = 2\left[\frac{1}{12}b^3 t + bt\left(\frac{b}{4}\right)^2\right] + \left[\frac{1}{12}t^3 2b + 2bt\left(\frac{b}{4}\right)^2\right] = \frac{5}{12}b^3 t + \frac{1}{6}bt^3$$

Assuming that $t << b$, and $t^3 \approx 0$

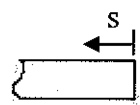
$$I_z = \frac{5}{12}b^3 t$$

Because symmetry, the shear flow of each section is,

$$q_s = -\frac{V_y Q_z}{I_z}$$

Where Q_z is, $Q_z = A^* y^*$ for each section.

For Section #1: $0 \le s \le b$

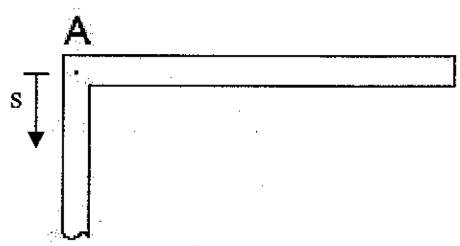


$$Q_{z,1} = st\left[\left(b - \frac{b}{4}\right) - \frac{s}{2}\right] = \frac{3}{4}tbs - \frac{1}{2}ts^2$$

$$q_1 = -\frac{V_y Q_{z,1}}{I_z} = -\frac{V_y\left(\frac{3}{4}tbs - \frac{1}{2}ts^2\right)}{\frac{5}{12}b^3 t} = -\frac{V_y\left(\frac{3}{4}bs - \frac{1}{2}s^2\right)}{\frac{5}{12}b^3}$$

$$q_{1\,(s=b)}^{A} = -\frac{V_y\left(\frac{3}{4}b^2 - \frac{1}{2}b^2\right)}{\frac{5}{12}b^3} = -\frac{3}{5}\frac{V_y}{b}$$

For Section #2: $0 \le s \le 2b$



$$Q_{z,2} = st\left[-\frac{b}{4}\right] = -\frac{1}{4}tbs$$

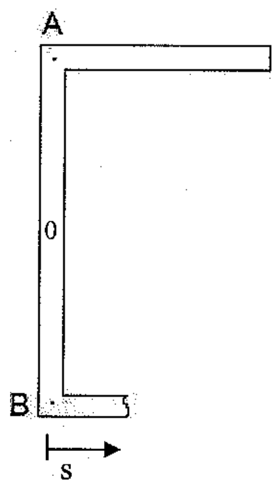
$$q_2 = q_1^A - \frac{V_y Q_{z,2}}{I_z} = -\frac{3}{5}\frac{V_y}{b} - \frac{V_y\left(-\frac{1}{4}tbs\right)}{\frac{5}{12}b^3 t} = -\frac{3}{5}\frac{V_y}{b} + \frac{V_y}{\frac{5}{3}b^2}s = \frac{3V_y}{5b^2}(s - b)$$

$$q^{0}_{2\,(s=b)} = \frac{3V_y}{5b^2}(b-b) = 0$$

As expected $q_2^0 = 0$!!!!!

$$q^{B}_{2\,(s=2b)} = \frac{3V_y}{5b^2}(2b-b) = \frac{3V_y}{5b}$$

For Section #3: $0 \le s \le b$



$$Q_{z,3} = st\left[\frac{s}{2} - \frac{b}{4}\right] = \frac{1}{2}ts^2 - \frac{1}{4}tbs$$

$$q_3 = q_2^B - \frac{V_y Q_{z,3}}{I_z} = \frac{3V_y}{5b} - \frac{V_y\left(\frac{1}{2}ts^2 - \frac{1}{4}tbs\right)}{\frac{5}{12}b^3 t} = \frac{3V_y}{5b} - \frac{12V_y\left(\frac{1}{2}s^2 - \frac{1}{4}bs\right)}{5b^3} = \frac{3V_y b^2 - 12V_y\left(\frac{1}{2}s^2 - \frac{1}{4}bs\right)}{5b^3}$$

$$q^{C}_{3\,(s=b)} = \frac{3V_y b^2 - 12V_y\left(\frac{1}{2}s^2 - \frac{1}{4}bs\right)}{5b^3} = 0$$

As expected $q_3^C = 0$!!!!!