

**MAE 3315**  
**Dr. Chan**  
**SOLUTION HW #6**

**Problem #1:**

First, take the Aluminum as a reference material.

$$A_{ii}^* = \frac{E_{ii}}{E_{al}} A_{ii} = \frac{15 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} A_{ii} = 1.5 A_{ii}$$

$$A_{al}^* = \frac{E_{al}}{E_{al}} A_{al} = \frac{10 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} A_{al} = 1 A_{al}$$

To calculate the centroid,

$$z_c^* = \frac{\sum A_i^* z_i}{\sum A_i^*} = \frac{1.5 A_{ii} (0.65 \text{ in}) + A_{al} (0.25 \text{ in})}{1.5 A_{ii} + A_{al}}$$

$$z_c^* = \frac{1.5(0.3 \text{ in})(1 \text{ in})(0.65 \text{ in}) + (0.5 \text{ in})(1 \text{ in})(0.25 \text{ in})}{1.5(0.3 \text{ in})(1 \text{ in}) + 1(0.5 \text{ in})(1 \text{ in})} = 0.439 \text{ in}$$

Finally, to calculate the moment of inertia,

$$I_y = \sum I_{yi}^* = \frac{E_{ii}}{E_{al}} (I_0 + Ad^2)_{ii} + (I_0 + Ad^2)_{al}$$

$$I_y = \frac{15 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} \left( \frac{1}{12} (1 \text{ in})(0.3 \text{ in})^3 + (1 \text{ in})(0.3 \text{ in})(0.439 - 0.65)^2 \right) + \left( \frac{1}{12} (1 \text{ in})(0.5 \text{ in})^3 + (1 \text{ in})(0.5 \text{ in})(0.439 - 0.25)^2 \right) = 5.169 \times 10^{-2} \text{ in}^4$$

**Problem #2:**

**Part a)** Centroid of the section.

Just as before, take the Aluminum as a reference material and substituting  $t = 0.04in$  and  $a = 12t = 0.48in$ ,

$$A_{ii}^* = \frac{E_{ii}}{E_{al}} A_{ii} = \frac{16 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} (2at) = \frac{16 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} (2(0.48in)(0.04in)) = 0.06144in^2$$

$$A_{al1}^* = \frac{E_{al}}{E_{al}} A_{al1} = \frac{10 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} at = 0.0192in^2$$

$$A_{al2}^* = \frac{E_{al}}{E_{al}} A_{al2} = \frac{10 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} (a - 2t)(t) = 0.016in^2$$

$$A_{al3}^* = \frac{E_{al}}{E_{al}} A_{al3} = \frac{10 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} at = 0.0192in^2$$

To calculate the centroid,

$$z_c^* = \frac{\sum A_i^* z_i}{\sum A_i^*} = \frac{(0.06144)(a + t/2) + (0.0192)(a/2) + (0.016)(a - t/2) + (0.0192)(a/2)}{0.06144 + 0.0192 + 0.016 + 0.0192}$$

$$z_c^* = \frac{(0.06144)(0.5) + (0.0192)(0.24) + (0.016)(0.46) + (0.0192)(0.24)}{0.11584}$$

$$z_c^* = 0.408in$$

**Part b)** Equivalent moment of inertia based on Aluminum.

$$I_y = \sum I_{yi}^* = \frac{E_{ii}}{E_{al}} (I_o + Ad^2)_{ii} + (I_o + Ad^2)_{al}$$

$$I_y = \frac{16 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} \left( \frac{1}{12} (2a)(t^3) + (2a)(t)(0.408 - 0.5)^2 \right) + \left( \frac{1}{12} (t)(a^3) + at(0.408 - 0.24)^2 \right) + \left( \frac{1}{12} (a - 2t)(t^3) + (a - 2t)(t)(0.408 - 0.46)^2 \right) + \left( \frac{1}{12} (t)(a^3) + at(0.408 - 0.24)^2 \right)$$

$$I_y = 2.395 \times 10^{-3} in^4$$

$$I_z = \sum I_{zi}^* = \frac{E_{ii}}{E_{al}} (I_0 + Ad^2)_{ii} + (I_0 + Ad^2)_{al}$$

$$I_z = \frac{16 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} \left( \frac{1}{12} (t)(2a)^3 + (2a)(t)(0)^2 \right) + \left( \frac{1}{12} (a)(t^3) + at(0.22)^2 \right) + \left( \frac{1}{12} (t)(a-2t)^3 + (a-2t)(t)(0)^2 \right) + \left( \frac{1}{12} (a)(t^3) + at(0.22)^2 \right)$$

$$I_z = 6.796 \times 10^{-3} \text{ in}^4$$

And because symmetry,

$$I_{yz} = 0$$

**Part c)** Max. bending stress and location.

To calculate the constants,

$$k_y^* = \frac{I_y^*}{I_y^* I_z^* - (I_{yz}^*)^2} = \frac{1}{I_z^*} = 147.1$$

$$k_z^* = \frac{I_z^*}{I_y^* I_z^* - (I_{yz}^*)^2} = \frac{1}{I_y^*} = 417.5$$

$$k_{yz}^* = \frac{I_{yz}^*}{I_y^* I_z^* - (I_{yz}^*)^2} = 0$$

Therefore,

$$\sigma_x = (147.1)(50)y + (417.5)(200)z$$

To calculate the neutral axis,

$$\tan \alpha = \frac{I_y M_z - I_{yz} M_y}{I_z M_y - I_{yz} M_z}$$

But  $I_{yz} = 0$ ,  $M_y = 200 \text{ lb} \cdot \text{in}$ , and  $M_z = 50 \text{ lb} \cdot \text{in}$  as a result,

$$\tan \alpha = \frac{I_y M_z}{I_z M_y}$$

$$\alpha = \text{ArcTan} \left( \frac{(2.395 \times 10^{-3})(50)}{(6.796 \times 10^{-3})(200)} \right)$$

$$\alpha = 5^\circ \text{ [CLOCKWISE]}$$

Therefore, there are only 2 possible locations for the maximum bending stress. The first one, the most top-right point; and the second one, the most bottom-left point.

For the first point  $y = 0.48in$  and  $z = 0.52in - 0.408in = 0.112in$

$$(\sigma_x)_{TOP-RIGHT} = (147.1)(50)(0.48) + (417.5)(200)(0.112)$$

$$(\sigma_x)_{TOP-RIGHT} = 12.86Ksi$$

For the second point  $y = -0.24in$  and  $z = -0.408in$

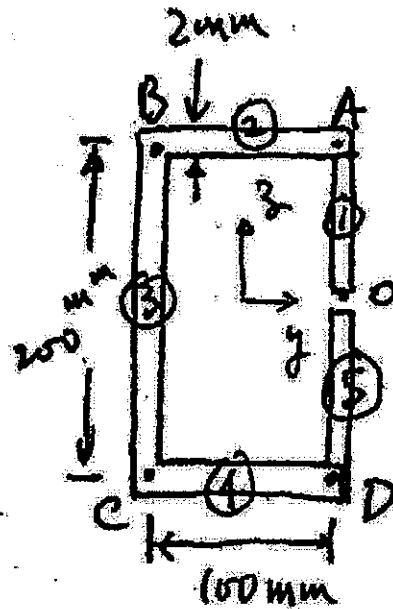
$$(\sigma_x)_{BOTTOM-LEFT} = (147.1)(50)(-0.24) + (417.5)(200)(-0.408)$$

$$\boxed{(\sigma_x)_{BOTTOM-LEFT} = -35.87Ksi}$$

Therefore, the bending stress is the highest at the bottom-left corner and it's in compression.

**Problem #3:**

For Figure 5.30:



The moment of inertia is,

$$I_y = \frac{1}{12} [(0.102)(0.202)^3 - 0.098(0.198)^3] = 6.667 \times 10^{-6} m^4$$

Because symmetry, the shear flow of each section is,

$$q_s = -\frac{V_z Q_y}{I_y}$$

Where  $Q_y$  is,  $Q_y = A^* z^*$  for each section.

**For Section #1:**  $0 \leq s \leq 0.1$  ( $0 \rightarrow A$ )

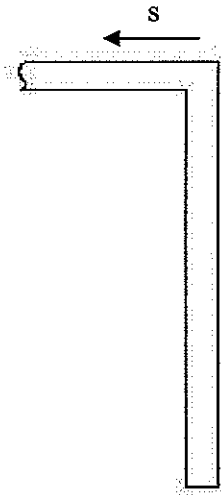


$$Q_1 = st \frac{s}{2} = 1 \times 10^{-3} s^2$$

$$q_1 = -\frac{V_z Q_1}{I_y} = -\frac{5000(1 \times 10^{-3} s^2)}{6.667 \times 10^{-6}} = -7.5 \times 10^5 s^2$$

$$q_1^A = -7.5 \times 10^5 (0.1)^2 = -7500$$

**For Section #2:**  $0 \leq s \leq 0.1$  ( $A \rightarrow B$ )



$$Q_2 = stz_c = s(0.002)(0.1) = 2 \times 10^{-4} s$$

$$q_2 = q_1^A - \frac{V_z Q_2}{I_y} = -7500 - \frac{5000(2 \times 10^{-4} s)}{6.667 \times 10^{-6}} = -7500 - 1.5 \times 10^5 s$$

$$q_2^B = -7500 - 1.5 \times 10^5 (0.1) = -22500$$

**For Section #3:**  $0 \leq s \leq 0.2$  ( $B \rightarrow C$ )

$$Q_3 = st \left( 0.1 - \frac{s}{2} \right) = 2 \times 10^{-4} s - 1 \times 10^{-3} s^2$$

$$q_3 = q_2^B - \frac{V_z Q_3}{I_y} = -22500 - \frac{5000(2 \times 10^{-4} s - 1 \times 10^{-3} s^2)}{6.667 \times 10^{-6}} = -22500 - 1.5 \times 10^5 s + 7.5 \times 10^5 s^2$$

$$q_3^C = -22500 - 1.5 \times 10^5 (0.2) + 7.5 \times 10^5 (0.2)^2 = -22500$$

NOTE:  $q_C = q_B!$

**For Section #4:**  $0 \leq s \leq 0.1$  ( $C \rightarrow D$ )

$$Q_4 = stz_c = s(0.002)(-0.1) = -2 \times 10^{-4} s$$

$$q_4 = q_3^C - \frac{V_z Q_4}{I_y} = -22500 - \frac{5000(-2 \times 10^{-4} s)}{6.667 \times 10^{-6}} = -22500 + 1.5 \times 10^5 s$$

$$q_4^D = -22500 + 1.5 \times 10^5 (0.1) = -7500$$

NOTE:  $q_D = q_A!$

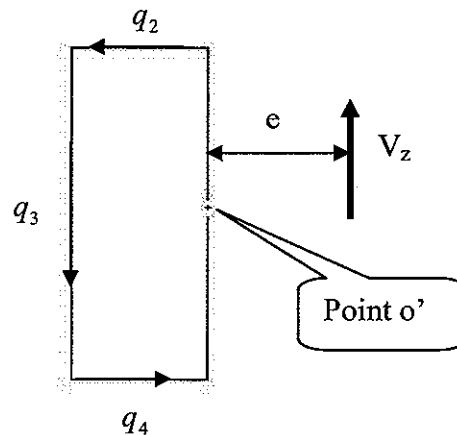
**For Section #5:**  $0 \leq s \leq 0.1$  ( $D \rightarrow E$ )

$$Q_5 = st \left( -0.1 + \frac{s}{2} \right) = -2 \times 10^{-4} s + 1 \times 10^{-3} s^2$$

$$q_5 = q_4^D - \frac{V_z Q_5}{I_y} = -7500 - \frac{5000(-2 \times 10^{-4} s + 1 \times 10^{-3} s^2)}{6.667 \times 10^{-6}} = -7500 + 1.5 \times 10^5 s - 7.5 \times 10^5 s^2$$

$$q_5^E = -7500 + 1.5 \times 10^5 (0.1) - 7.5 \times 10^5 (0.1)^2 = 0$$

To calculate the Shear Center, we can take summation of moments at middle-right point (point o'). In that way,  $q_1$  and  $q_5$  will not contribute to the total summation of moments.



Therefore,

$$V_z e = \left( \int q_2 ds \right) 0.1 + \left( \int q_3 ds \right) 0.1 + \left( \int q_4 ds \right) 0.1$$

$$V_z e = \left( \int_0^{0.1} (-7500 - 1.5 \times 10^5 s) ds \right) 0.1 + \left( \int_0^{0.2} (-22500 - 1.5 \times 10^5 s + 7.5 \times 10^5 s^2) ds \right) 0.1 +$$

$$+ \left( \int_0^{0.1} (-22500 + 1.5 \times 10^5 s) ds \right) 0.1$$

$$V_z e = \left( -7500s - 1.5 \times 10^5 \frac{s^2}{2} \right) \Big|_0^{0.1} 0.1 + \left( -22500s - 1.5 \times 10^5 \frac{s^2}{2} + 7.5 \times 10^5 \frac{s^3}{3} \right) \Big|_0^{0.2} 0.1 +$$

$$+ \left( -22500s + 1.5 \times 10^5 \frac{s^2}{2} \right) \Big|_0^{0.1} 0.1$$

$$V_z e = \left[ -7500(0.1) - 1.5 \times 10^5 \frac{(0.1)^2}{2} \right] 0.1 + \left[ -22500(0.2) - 1.5 \times 10^5 \frac{(0.2)^2}{2} + 7.5 \times 10^5 \frac{(0.2)^3}{3} \right] 0.1 +$$

$$+ \left[ -22500(0.1) + 1.5 \times 10^5 \frac{(0.1)^2}{2} \right] 0.1$$

$$V_z e = -850$$

$$5000e = -850$$

$$e = \frac{-850}{5000} = -0.170m$$

**NOTE:** Please, realize that “e” does NOT depend on the value of  $V_z$  (it cancels out).

**For Figure 5.32:**

Assuming that the bottom-left stringer is the origin,

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{(2h)(2A_0) + (2h)(2A_0)}{2A_0 + A_0 + A_0 + 2A_0} = \frac{4}{3}h$$

$$z_c = \frac{\sum z_i A_i}{\sum A_i} = \frac{(h)(2A_0) + (h)(A_0)}{2A_0 + A_0 + A_0 + 2A_0} = \frac{1}{2}h$$

To calculate the moment of inertia,

$$I_y = \sum_i A_i z_i^2 = (2A_0)(h/2)^2 + (A_0)(h/2)^2 + (A_0)(h/2)^2 + (2A_0)(h/2)^2 = \frac{3}{2}A_0 h^2$$

Again, because symmetry, the shear flow of each section is,

$$q_s = -\frac{V_z Q_y}{I_y}$$

**For Section #1:** (The upper horizontal thin sheet).

$$Q_1 = (2A_0)(h/2) = A_0 h$$

$$q_1 = -\frac{V_z Q_1}{I_y} = -\frac{V_z (A_0 h)}{\frac{3}{2}A_0 h^2} = -\frac{2}{3} \frac{V_z}{h}$$

**For Section #2:** (The semicircular thin sheet).

$$Q_2 = (A_0)(h/2) = \frac{A_0 h}{2}$$

$$q_2 = q_1 - \frac{V_z Q_2}{I_y} = -\frac{2}{3} \frac{V_z}{h} - \frac{V_z \left( \frac{A_0 h}{2} \right)}{\frac{3}{2}A_0 h^2} = -\frac{2}{3} \frac{V_z}{h} - \frac{1}{3} \frac{V_z}{h} = -\frac{V_z}{h}$$

**For Section #3:** (The lower horizontal thin sheet).

$$Q_3 = (A_0)(-h/2) = -\frac{A_0 h}{2}$$

$$q_3 = q_2 - \frac{V_z Q_3}{I_y} = -\frac{V_z}{h} - \frac{V_z \left( -\frac{A_0 h}{2} \right)}{\frac{3}{2} A_0 h^2} = -\frac{V_z}{h} + \frac{1}{3} \frac{V_z}{h} = -\frac{2}{3} \frac{V_z}{h}$$

Taking summation of moments at the center of the vertical line that defined the semicircle,

$$V_z e = q_1 (2h) \left( \frac{h}{2} \right) + q_2 (2\bar{A}_2) + q_3 (2h) \left( \frac{h}{2} \right)$$

$$V_z e = -\frac{2}{3} V_z h - \frac{V_z}{h} \left( 2 \frac{1}{2} \pi \left( \frac{h}{2} \right)^2 \right) - \frac{2}{3} V_z h$$

$$V_z e = -\frac{4}{3} V_z h - \frac{1}{4} V_z h \pi$$

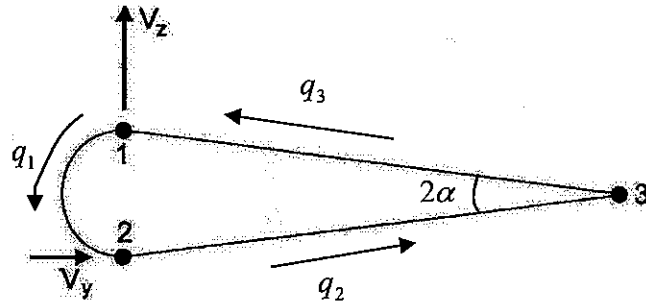
As we can see,  $V_z$  cancels out and " $e$ " is independent of it, as we said before.

$$e = h \left[ -\frac{4}{3} - \frac{1}{4} \pi \right]$$

$$e = -2.12h$$

**Problem #4:**

**Part a)**



$$\sum F_y = 0: \quad q_2 l_{23} \cos(\alpha) = q_3 l_{31} \cos(\alpha)$$

And since  $l_{23} = l_{31}$ ,

$$q_2 = q_3$$

$$\sum F_z = 0 \quad V_z = -q_1(0.2) + q_2 l_{23} \sin(\alpha) + q_3 l_{31} \sin(\alpha)$$

Since  $l_{23} \sin(\alpha) = 0.1$  and  $q_2 = q_3$

$$V_z = -q_1(0.2) + q_2(0.2)$$

$$V_z = 0.2(q_2 - q_1)$$

Taking summation of moments at point 3,

$$-V_z(0.8) = 2\bar{A}q_1 = 2q_1 \left( \frac{1}{2} \pi (0.1)^2 + \frac{1}{2} (0.8)(0.2) \right)$$

$$-(5000)(0.8) = q_1(0.01\pi + 0.16)$$

$$q_1 = -20897 \text{ N/m} = -20.9 \text{ kN/m}$$

Then,

$$q_2 = q_1 - \frac{V_z Q_2}{I_y}$$

Where,

$$I_y = (0.1)^2 A + (-0.1)^2 A = 2 \times 10^{-5} m^4$$

$$Q_2 = -0.1A = -0.0001 m^2$$

$$q_2 = q_1 - \frac{V_z(-0.0001)}{2 \times 10^{-5}} = -20897 - \frac{5000(-0.0001)}{2 \times 10^{-5}} = 4103 N/m = 4.10 kN/m$$

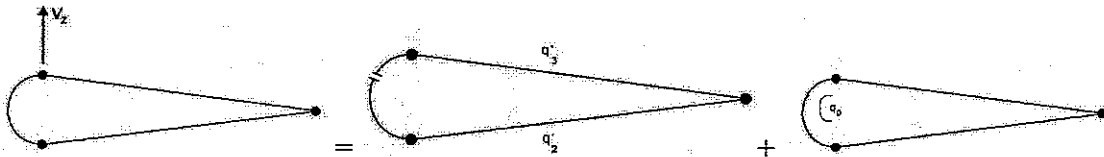
$$q_3 = q_2 = 4.10 kN/m$$

To calculate the twist angle,

$$\theta = \frac{1}{2GA} \oint q \frac{ds}{t} = \frac{1}{2(27 \times 10^9)(0.09571)(0.001)} [q_2 l_{23} + q_3 l_{31} + q_1 arc_{12}]$$

$$\theta = 0.1935 \times 10^{-6} [4103 \sqrt{0.1^2 + 0.8^2} + 4103 \sqrt{0.1^2 + 0.8^2} - 20897(0.1\pi)]$$

$$\theta = 9.8710^{-6} rad/m = 5.65 \times 10^{-4} deg/m$$



$$\sum F_y = 0 \quad \Rightarrow \quad q_2 = q_3$$

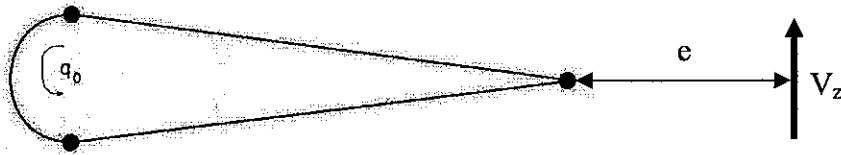
$$q_2 = -\frac{V_z Q_2}{I_y} = -\frac{(5000)(0.1)(0.001)}{2 \times 10^{-5}} = -2.5 \times 10^4 N/m$$

$$\theta = \frac{1}{2GA} \oint q \frac{ds}{t} = 0 \quad \Rightarrow \quad \oint q ds = 0$$

$$q_0(0.1\pi) + (q_0 + q_2)l_{23} + (q_0 + q_3)l_{31} = 0$$

$$q_0(0.1\pi) + 2(q_0 - 2.5 \times 10^4)\sqrt{0.1^2 + 0.8^2} = 0$$

$$q_0 = \frac{(5.0 \times 10^4)(0.806)}{0.1\pi + 2(0.806)} = 2.092 \times 10^4 \text{ N/m}$$



$$V_z e = q_0 2\bar{A}$$

$$e = \frac{(2.092 \times 10^4)(0.1914)}{5000} = 0.8 \text{ m}$$

$$\sum F_y = 0: \quad -\int_0^\pi q_1 ds \cos \theta + q_2 l_{23} \cos(\alpha) - q_3 l_{31} \cos(\alpha) = 0$$



$$\sum F_z = 0 \quad -\int_0^\pi (q_1 ds) \sin \theta + q_2 l_{23} \sin(\alpha) + q_3 l_{31} \sin(\alpha) + V_z = 0$$

$$-\int_0^\pi (q_1 R \sin \theta d\theta) + 2q_2 0.1 + V_z = 0$$

$$q_1(0.1)\text{Cos}\theta\Big|_0^\pi + 2q_2 \cdot 0.1 = V_z$$

$$q_1(0.1)\text{Cos}(\pi) - q_1(0.1)\text{Cos}(0) + 2q_2 \cdot 0.1 = V_z$$

$$-q_1(0.1) - q_1(0.1) + 2q_2 \cdot 0.1 = V_z$$

$$-2q_1(0.1) + 2q_2 \cdot 0.1 = V_z$$

$$V_z = 0.2(q_2 - q_1) \quad \text{The equation is checked!}$$

**Part b)**

$$\sum F_y = 0 \quad \Rightarrow \quad V_y = q_2 l_{23} \text{Cos}(\alpha) - q_3 l_{31} \text{Cos}(\alpha)$$

$$\text{Since } l_{23} \text{Cos}(\alpha) = 0.80$$

$$10,000 = q_2 \cdot 0.8 - q_3 \cdot 0.8 \quad \text{Equation \#1}$$

$$\sum F_z = 0 \quad \Rightarrow \quad V_z = q_2 l_{23} \text{Sin}(\alpha) + q_3 l_{31} \text{Sin}(\alpha) - \int_0^\pi (q_1 R \text{Sin}\theta d\theta)$$

$$\text{Once again, } l_{23} \text{Sin}(\alpha) = l_{31} \text{Sin}(\alpha) = 0.1,$$

$$5000 = q_2 \cdot 0.1 + q_3 \cdot 0.1 + q_1(0.1)\text{Cos}\theta\Big|_0^\pi$$

$$5000 = q_2 \cdot 0.1 + q_3 \cdot 0.1 - 2q_1(0.1) \quad \text{Equation \#2}$$

Taking summation of moments at point 3,

$$V_y \cdot 0.1 - V_z \cdot 0.8 = 2\bar{A}q_1 = 2q_1 \left( \frac{1}{2} \pi (0.1)^2 + \frac{1}{2} (0.8)(0.2) \right)$$

$$(10,000)0.1 - (5,000)0.8 = q_1(0.01\pi + 0.16) \quad \text{Equation \#3}$$

Solving Equation #3,

$$q_1 = \frac{(10,000)0.1 - (5,000)0.8}{(0.01\pi + 0.16)} = -15672 \text{ N/m} = -15.7 \text{ kN/m}$$

Solving Equation #1 and #2 together,

$$q_2 = 15578 \text{ N/m} = 15.6 \text{ kN/m}$$

$$q_3 = 3078 \text{ N/m} = 3.1 \text{ kN/m}$$

$$\theta = \frac{1}{2GA} [q_1 \text{arc}_{12} + q_2 l_{23} + q_3 l_{31}]$$

$$\theta = \frac{1}{2(27 \times 10^9)(0.09571)(0.001)} [-15672(0.1\pi) + 15578\sqrt{0.1^2 + 0.8^2} + 3078\sqrt{0.1^2 + 0.8^2}]$$

$$\theta = 0.1935 \times 10^{-6} [-15672(0.1\pi) + 15578\sqrt{0.1^2 + 0.8^2} + 3078\sqrt{0.1^2 + 0.8^2}]$$

$$\theta = 1.9577 \times 10^{-3} \text{ rad/m}$$

**Problem #5:**

Assuming that the bottom-left stringer (#2) is the origin,

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{(80)(10) + (80)(10)}{15 + 15 + 10 + 10} = 32 \text{ cm}$$

$$z_c = \frac{\sum z_i A_i}{\sum A_i} = \frac{(40)(15) + (20)(10)}{15 + 15 + 10 + 10} = 16 \text{ cm}$$

The moment of inertia is,

$$I_y = \sum_i A_i z_i^2 = (15)(40 - 16)^2 + (15)(16)^2 + (10)(16)^2 + (10)(20 - 16)^2 = 15,200 \text{ cm}^4$$

$$I_z = \sum_i A_i y_i^2 = (15)(32)^2 + (15)(32)^2 + (10)(80 - 32)^2 + (10)(80 - 32)^2 = 76,800 \text{ cm}^4$$

$$I_{yz} = \sum_i A_i y_i z_i = (15)(-32)(40 - 16) + (15)(-32)(-16) + (10)(80 - 32)(-16) + (10)(80 - 32)(20 - 16) = -9,600 \text{ cm}^4$$

The constants  $k_y$ ,  $k_z$ , and  $k_{yz}$  are now determined,

$$k_y = \frac{I_y}{I_y I_z - I_{yz}^2} = \frac{15,200}{(15,200)(76,800) - (-9,600)^2} = 1.4137 \times 10^{-5} \text{ cm}^4$$

$$k_z = \frac{I_z}{I_y I_z - I_{yz}^2} = \frac{76,800}{(15,200)(76,800) - (-9,600)^2} = 7.1429 \times 10^{-5} \text{ cm}^4$$

$$k_{yz} = \frac{I_{yz}}{I_y I_z - I_{yz}^2} = \frac{-9,600}{(15,200)(76,800) - (-9,600)^2} = -8.9286 \times 10^{-6} \text{ cm}^4$$

The shear flow is,

$$q_s = -(k_y V_y - k_{yz} V_z) \rho_z - (k_z V_z - k_{yz} V_y) \rho_y$$

$$q_s = (k_{yz} V_z) \rho_z - (k_z V_z) \rho_y$$

Therefore,

$$q_{12} = (-8.9286 \times 10^{-6})(5000)(-32)(15) - (7.1429 \times 10^{-5})(5000)(40 - 16)(15) = -107.14 \text{ N/cm}$$

$$q_{23} = q_{12} + (-8.9286 \times 10^{-6})(5000)(-32)(15) - (7.1429 \times 10^{-5})(5000)(-16)(15) = 0 \text{ N/cm}$$

$$q_{34} = q_{23} + (-8.9286 \times 10^{-6})(5000)(80 - 32)(10) - (7.1429 \times 10^{-5})(5000)(-16)(10) = 35.71 \text{ N/cm}$$

Taking summation of moments at point 2,

$$V_z e_y = (80 \text{ cm})(35.71 \text{ N/cm})(20 \text{ cm})$$

$$e_y = \frac{(80 \text{ cm})(35.71 \text{ N/cm})(20 \text{ cm})}{5,000 \text{ N}} = 11.43 \text{ cm}$$

$$y_{sc} = 11.43 - 32 = -20.57 \text{ cm}$$

To calculate  $z_{sc}$  we need to assume a  $V_y$  and  $V_z = 0$ . In this case, the shear flow becomes

$$q_s = -(k_y V_y) Q_z + (k_{yz} V_y) Q_y$$

$$q_{12} = -(1.4137 \times 10^{-5}) V_y (-32)(15) + (-8.9286 \times 10^{-6}) V_y (40 - 16)(15) = 3.5714 \times 10^{-3} V_y$$

$$q_{23} = q_{12} + (1.4137 \times 10^{-5}) V_y (-32)(15) + (-8.9286 \times 10^{-6}) V_y (-16)(15) = -1.0714 \times 10^{-3} V_y$$

$$q_{34} = q_{23} + (1.4137 \times 10^{-5}) V_y (80 - 32)(10) + (-8.9286 \times 10^{-6}) V_y (-16)(10) = 7.1429 \times 10^{-3} V_y$$

Once again, taking summation of moments at point 2,

$$V_y e_z = (80)(7.1429 \times 10^{-3} V_y)(20)$$

As usually,  $V_y$  cancels out,

$$e_z = (80)(7.1429 \times 10^{-3})(20) = 11.43 \text{ cm}$$

$$z_{sc} = -16 - 11.43 = -27.43 \text{ cm}$$