MAE3315
HW # 8. (SOLUTION)

Problem 8-1

A straight round steel shaft is subjected to a torque of 6,000 lb-in. The material has yielding strength of 60 Ksi and a safety factor of 2. Determine the minimum required diameter that will not give a yielding failure by using.

a) Maximum principal stress criterion
b) Maximum shear stress criterion
c) Von-Mises failure criterion

Solution:

For a round shaft under torque,

$$\tau_{MAX} = \frac{Tr}{J} = \frac{Tr}{\frac{\pi r^4}{2}} = \frac{2T}{\pi r^3} = \frac{12{,}000}{\pi}\frac{1}{r^3}$$

a) Maximum principal stress criterion

First we need to find the principal stresses. At the surface of the shaft, there is pure shear acting,

$$[\sigma] = \begin{bmatrix} 0-\sigma & \tau_{MAX} \\ \tau_{MAX} & 0-\sigma \end{bmatrix}$$

$$\sigma^2 - \tau_{MAX}^2 = 0$$

$$\sigma = \pm\sqrt{\tau_{MAX}^2}$$

$$\sigma_{1,2} = \pm\frac{12{,}000}{\pi}\frac{1}{r^3}$$

$$\sigma_1 = +\frac{12{,}000}{\pi}\frac{1}{r^3} \qquad \text{and} \qquad \sigma_2 = -\frac{12{,}000}{\pi}\frac{1}{r^3}$$

On the other hand,

$$S.F. = \frac{\sigma_1^{allow}}{\sigma_1} \quad \rightarrow \quad 2\sigma_1 = \sigma_1^{allow} \quad \rightarrow \quad \sigma_1^{allow} = 2\left(\frac{12{,}000}{\pi}\frac{1}{r^3}\right) \quad \rightarrow \quad \sigma_1^{allow} = \frac{24{,}000}{\pi}\frac{1}{r^3}$$

$$S.F. = \frac{\sigma_2^{allow}}{\sigma_2} \rightarrow 2\sigma_2 = \sigma_2^{allow} \rightarrow \sigma_2^{allow} = 2\left(-\frac{12{,}000}{\pi}\frac{1}{r^3}\right) \rightarrow \sigma_2^{allow} = -\frac{24{,}000}{\pi}\frac{1}{r^3}$$

Finally, when,

$$\left|\sigma_1^{allow}\right| \leq \sigma_Y \rightarrow \frac{24{,}000}{\pi}\frac{1}{r^3} = 60{,}000 psi \rightarrow r = 0.50 in \rightarrow \boxed{d_{\min} = 1.01 in}$$

$$\left|\sigma_2^{allow}\right| \leq \sigma_Y \rightarrow \frac{24{,}000}{\pi}\frac{1}{r^3} = 60{,}000 psi \rightarrow r = 0.50 in \rightarrow \boxed{d_{\min} = 1.01 in}$$

b) Maximum shear stress criterion

There are 3 conditions,

$$\left|\sigma_1^{allow} - \sigma_2^{allow}\right| \leq \sigma_Y$$
$$\left|\sigma_1^{allow}\right| \leq \sigma_Y$$
$$\left|\sigma_1^{allow}\right| \leq \sigma_Y$$

For the last two conditions we already know the minimum diameter possible to prevent yielding is d=1.01inches. However, for the first condition,

$$\left|\left(\frac{24{,}000}{\pi}\frac{1}{r^3}\right) - \left(-\frac{24{,}000}{\pi}\frac{1}{r^3}\right)\right| = 60{,}000 psi$$

$$\frac{48{,}000}{\pi}\frac{1}{r^3} = 60{,}000 psi$$

$$r = \sqrt[3]{\frac{48{,}000}{60{,}000}\frac{1}{\pi}}$$

$$r = 0.63 in$$

$$\boxed{d_{\min} = 1.27 in}$$

Therefore, the minimum diameter for the shaft has to be 1.27 inches.

c) Von-Mises failure criterion

$$\left(\sigma_1^{allow}\right)^2 + \left(\sigma_2^{allow}\right)^2 - \left(\sigma_1^{allow}\right)\left(\sigma_2^{allow}\right) \le \left(\sigma_Y\right)^2$$

$$\left(\frac{24{,}000}{\pi}\frac{1}{r^3}\right)^2 + \left(-\frac{24{,}000}{\pi}\frac{1}{r^3}\right)^2 - \left(\frac{24{,}000}{\pi}\frac{1}{r^3}\right)\left(-\frac{24{,}000}{\pi}\frac{1}{r^3}\right) = \left(60{,}000\right)^2$$

$$3\left(\frac{24{,}000}{\pi}\frac{1}{r^3}\right)^2 = \left(60{,}000\right)^2$$

$$r = 0.60in$$

$$\boxed{d_{\min} = 1.21in}$$

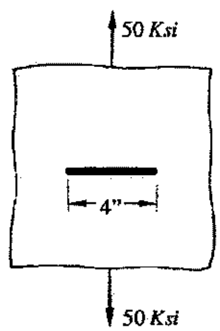
Problem 8-2

A large thin steel plate is loaded as shown below in each case. The material properties of the plate is given as

$$E = 30\times10^6\ psi\ ,\quad \nu = 0.3\ ,\quad \sigma_{ult} = 82\ ksi$$

$$\sigma_Y = 47\ ksi\ ,\quad K_c = 110\ ksi\sqrt{in}$$

An infinite thin plate has a 4" crack at the center. Determine:



(a) the max. permissible load under this crack length

(b) the max. permissible crack length if the plate is subjected to 50 ksi.

Solution:

Part a)

$$K_C = \sigma_c\sqrt{\pi a}$$

$$110ksi\sqrt{in} = \sigma_c\sqrt{\pi 2in}$$

$$\sigma_c = \frac{110ksi\sqrt{in}}{\sqrt{\pi 2in}} = \boxed{43.88ksi}$$

Part b)

$$K_C = \sigma_o\sqrt{\pi a_c}$$

$$110ksi\sqrt{in} = 50ksi\sqrt{\pi a_c}$$

$$a_c = \left(\frac{110ksi\sqrt{in}}{50ksi\sqrt{\pi}}\right)^2 = 1.54in$$

$$\boxed{\text{Crack length } = 3.08in}$$