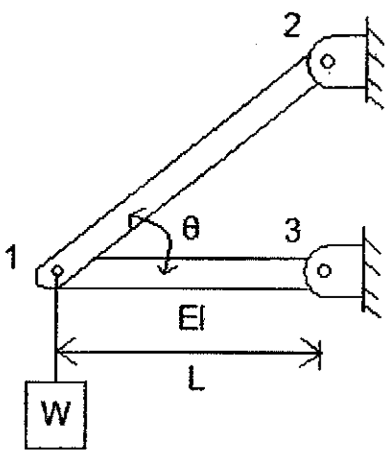
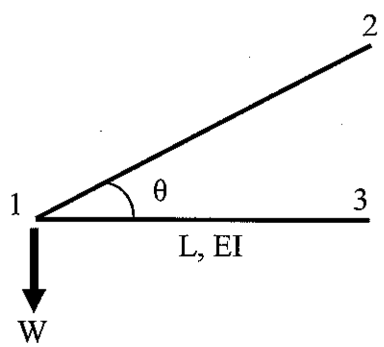
MAE3315
HW # 11. (SOLUTION)

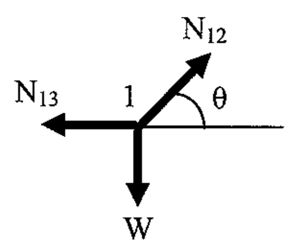
Problem 11-1 (Refer to Problem 7.1 in the text, Page 266)

The truss structure consists of two bars connected by a pin-joint (which allows free rotation of the bars.) The other ends of the bars are hinged as shown in the figure. A weight W is hung at the joint. Find the maximum weight (in terms of EI, L and θ) the truss can sustain before buckling.



Solution:





Note: Bar 1-2 is on tension, so it won't buckle. On the bar 1-3 is on compression, so we need to check if it buckles.

$$N_{12} Sin\theta = W$$

$$N_{13} = N_{12} Cos\theta = \frac{W}{Sin\theta} Cos\theta = W Cot\theta$$

Now the free-body diagram for the bar 1-3,

$$P_{CR} = \frac{\pi^2 EI}{L_{eff}^2} = W Cot\theta$$

However, $k = 1 \quad \Rightarrow \quad L_{eff} = L$

$$\boxed{W = \frac{\pi^2 EI}{L^2} Tan\theta}$$

Problem 11-2

A hollow circular steel column with Young's modulus, $E=30x10^3$ Ksi and yield strength, $\sigma_Y=220$ Ksi is simply supported over a length 10 ft. The inner and outer diameters of the cross-section are 3 in and 4 in., respectively. Determine:

a) The critical slenderness ratio
b) The critical buckling load
c) If a roller supports are added at the midpoint, what would be the new critical buckling load?

Solution:

Part a) Slenderness ratio

$$I=\frac{\pi}{64}\left(d_o^4-d_i^4\right)=\frac{\pi}{64}\left[(4in)^4-(3in)^4\right]=8.59in^4$$

$$A=\frac{\pi}{4}\left(d_o^2-d_i^2\right)=\frac{\pi}{4}\left[(4in)^2-(3in)^2\right]=5.50in^2$$

$$r=\sqrt{\frac{I}{A}}=\sqrt{\frac{8.59in^4}{5.50in^2}}=1.25in$$

$$\frac{L_{eff}}{r}=\frac{k\cdot L}{r}=\frac{(1)(10ft)\left(12\frac{in}{ft}\right)}{1.25in}=\boxed{96}$$

Part b) Critical buckling load

$$P_{CR}=\frac{\pi^2 EI}{L_{eff}^2}=\frac{\pi^2\left(30x10^6\,psi\right)\left(8.59in^4\right)}{\left[(10ft)\left(12\frac{in}{ft}\right)\right]^2}=\boxed{176.6kips}$$

Checking validity of Euler equation,

$$\left(\frac{L_{eff}}{r}\right)_C=\sqrt{\frac{\pi^2 E}{0.5\sigma_y}}=\sqrt{\frac{\pi^2\left(30x10^6\,psi\right)}{0.5\left(220x10^3\,psi\right)}}=51.88$$

$$\left(\frac{L_{eff}}{r}\right)_C=51.88<96=\frac{L_{eff}}{r}\quad \text{Euler equation is valid}$$

Part c) Critical buckling load if roller added at midpoint

$$L_{eff} = \frac{L}{2} = 5\,ft = 60\,inches$$

$$\frac{L_{eff}}{r} = \frac{60in}{1.25in} = 48$$

$$\left(\frac{L_{eff}}{r}\right)_C = 51.88 > 48 = \frac{L_{eff}}{r}$$ Euler equation is <u>NOT</u> valid

Therefore, Short Column theory should be used,

$$\sigma_C = \sigma_y - \frac{\sigma_y}{2}\frac{\left(\frac{L_{eff}}{r}\right)^2}{\left(\frac{L_{eff}}{r}\right)_C^2}$$

$$\sigma_C = 220Ksi - \frac{220Ksi}{2}\frac{(48)^2}{(51.88)^2}$$

$$\sigma_C = 125.8Ksi$$

$$P_C = A\sigma_C$$

$$P_C = (5.50in^2)(125{,}800\,psi)$$

$$\boxed{P_C = 691.9kips}$$

Problem 11-3

A square steel column with a solid cross section is pinned at both its base and top. The column is 5 m in height and supports a load of $4.5x10^6$N. The modulus of elasticity of the steel is 210 GPa and the yield stress is 1520 MPa. What is the minimum cross-section size to avoid buckling with a safety factor of 1.5?

Solution:

Because it's a pined-pined column, $k = 1$.

$$L_{eff} = k \cdot L = (1)(5m) = 5m$$

The applied load and safety factor,

$$P_C = 1.5(4.5x10^6 N)$$

$$P_C = 6.75x10^6 N$$

Euler equation,

$$P_C = \frac{\pi^2 EI}{L_{eff}^2}$$

$$6.75x10^6 N = \frac{\pi^2 (210x10^9 Pa) I}{(5m)^2}$$

$$I = \frac{6.75x10^6 N (5m)^2}{\pi^2 (210x10^9 Pa)}$$

$$I = 8.14188x10^{-5} m^4$$

Because the cross-section is square, we know that,

$$I = \frac{1}{12} b^4 = 8.14188x10^{-5} m^4$$

$$\boxed{b = 18\,cm}$$

Checking validity of Euler equation,

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12}b^4}{b^2}} = \frac{b}{\sqrt{12}} = 5.1037x10^{-2}\,m$$

$$\frac{L_{eff}}{r} = \frac{5m}{5.1037x10^{-2}\,m} = 98$$

On the other hand,

$$\left(\frac{L_{eff}}{r}\right)_C = \sqrt{\frac{\pi^2 E}{0.5\sigma_y}} = \sqrt{\frac{\pi^2(210x10^9\,Pa)}{0.5(1520x10^6\,Pa)}} = 52.22$$

$$\left(\frac{L_{eff}}{r}\right)_C = 52.22 < 98 = \left(\frac{L_{eff}}{r}\right)$$

Therefore, the Euler equation is valid.