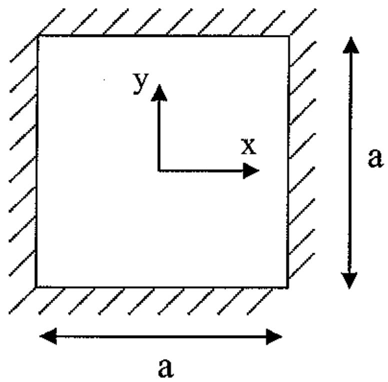
MAE3315
Supplement 2

A Constrained Plate under a Thermal Gradient

Square Plate:



As we know,

$$\varepsilon^{TOTAL} = \varepsilon^{MECHANICAL} + \varepsilon^{THERMAL}$$

In this case, the total strain is equal to "zero" because of the constrain.

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \alpha\Delta T = 0$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} + \alpha\Delta T = 0$$

From these two equations, we get that,

$$\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \alpha\Delta T = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} + \alpha\Delta T$$

$$\sigma_x - \nu\sigma_y = \sigma_y - \nu\sigma_x$$

$$(1+\nu)\sigma_x = (1+\nu)\sigma_y$$

$$\boxed{\sigma_x = \sigma_y}$$

So, both stresses will be the same! Now, substituting back,

$$\frac{\sigma_x}{E} - \nu\frac{\sigma_x}{E} + \alpha\Delta T = 0$$

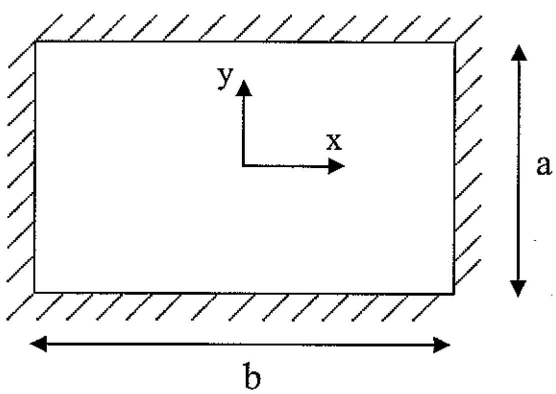
$$\frac{\sigma_x}{E} - \nu\frac{\sigma_x}{E} = -\alpha\Delta T$$

$$(1-\nu)\sigma_x = -E\alpha\Delta T$$

$$\boxed{\sigma_x = -\frac{E\alpha\Delta T}{(1-\nu)} = \sigma_y}$$

So the stresses are independent of the geometry. That is, σ_x is the same for every point in the square plate; the same happens for σ_y.

Rectangular Plate:



Once again, the total strain will be equal to zero. Therefore,

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \alpha\Delta T = 0$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} + \alpha\Delta T = 0$$

Solving again these two equations, we get that,

$$\boxed{\sigma_x = \sigma_y}$$

$$\boxed{\sigma_x = -\frac{E\alpha\Delta T}{(1-\nu)} = \sigma_y}$$

How is this possible?

Well, under these conditions in the rectangular plate, the horizontal (σ_x) and vertical (σ_y) stresses will be the same. However, these stresses are acting on different areas; therefore, the forces are different. Remember that,

$$P_x = \sigma_x A$$

Therefore, for the Square Plate,

$$P_x^{SQUARE} = \sigma_x a\Delta z$$

On the other hand, for the Rectangular Square,

$$P_x^{RECT} = \sigma_x a\Delta z = \left(-\frac{E\alpha\Delta T}{(1-\nu)}\right)a\Delta z$$

$$P_y^{RECT} = \sigma_y b\Delta z = \left(-\frac{E\alpha\Delta T}{(1-\nu)}\right)b\Delta z$$

Finally,

$$\boxed{P_x^{RECT} = \frac{a}{b}P_y^{RECT}}$$

Note: Remember that both forces are in compression; that is why they are both negative.