MAE3315
Supplement 3.1

Integration of the torque for a circular section

$$T = 2C\iint_A \left[\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 - 1\right] dA$$

$$T = 2C\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \left[\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 - 1\right] dx\,dy$$

$$T = 2C\int_{-a}^{a} \left(\left[\frac{1}{3}\frac{x^3}{a^2} + \frac{y^2}{a^2}x - x \right]_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \right) dy$$

$$T = 2C\int_{-a}^{a} \left(\frac{4}{3}\frac{\left(y^2 - a^2\right)\sqrt{a^2 - y^2}}{a^2} \right) dy$$

$$T = 2C\left[\left(-\frac{1}{3}\frac{\left(a^2 - y^2\right)^{\frac{3}{2}}}{a^2} - \frac{1}{2}\sqrt{a^2 - y^2} \right) y - \frac{1}{2}a^2\, ArcTan\left(\frac{y}{\sqrt{a^2 - y^2}} \right) \right]_{-a}^{a}$$

$$T = 2C\left[\left(-\frac{1}{2}a^2\, ArcTan(\infty) \right) - \left(-\frac{1}{2}a^2\, ArcTan(-\infty) \right) \right]$$

$$T = 2C\left[\left(-\frac{1}{2}a^2\frac{\pi}{2} \right) - \left(-\frac{1}{2}a^2\left(-\frac{\pi}{2} \right) \right) \right]$$

$$T = 2C\left[\left(-\frac{1}{4}a^2\pi \right) - \left(\frac{1}{4}a^2\pi \right) \right]$$

$$T = 2C\left[-\frac{1}{2}a^2\pi \right]$$

$$\boxed{T = -Ca^2\pi}$$

Same answer as we found in class!