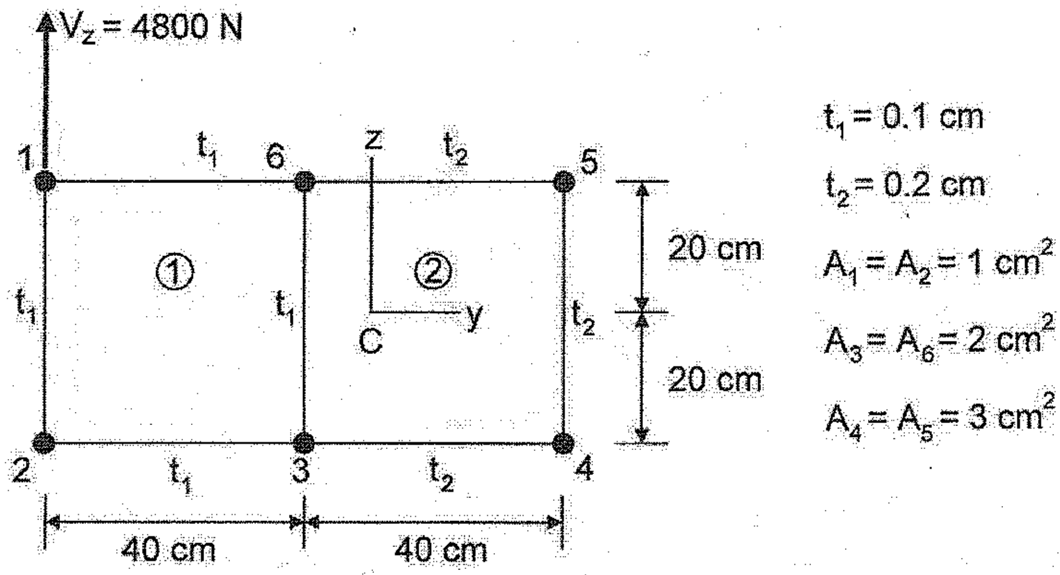
MAE3315
Dr. Chan
SUPPLEMENTS 5-3

Example 5.9 (From the book, p. 174)

The two-cell box beam section shown in the figure is symmetrical about the y-axis. Assume that the sheets are ineffective in bending.



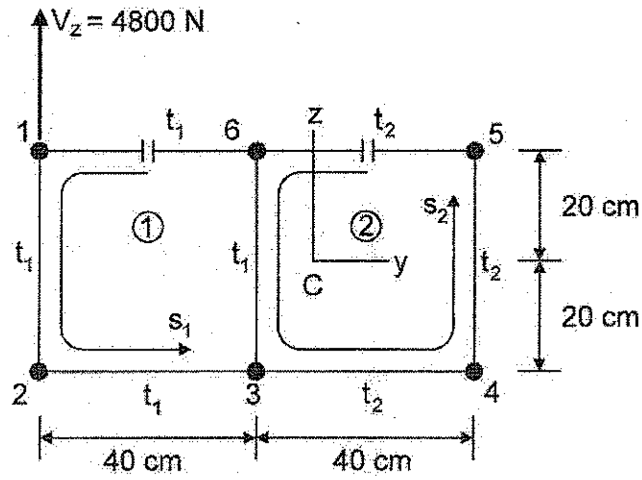
Solution:

First, we need to calculate the centroidal y and z axes. First, $\bar{y}_c$ is not needed, so it will not be calculated. On the other hand, because symmetry, $\bar{z}_c = 20cm$. Then, to calculate the inertia,

$$I_y = \sum_i A_i z_i^2 = (1cm^2)(20cm)^2 + (1cm^2)(-20cm)^2 + (2cm^2)(20cm)^2 + (2cm^2)(-20cm)^2 +$$
$$+ (3cm^2)(20cm)^2 + (3cm^2)(-20cm)^2$$

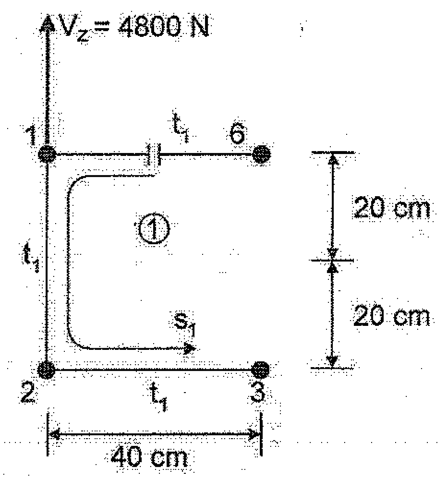
$$I_y = 4{,}800cm^4$$

Then, "cut" the sheets between stringers 1 and 6 and between 5 and 6, and set up the contours s_1 and s_2 as shown in the figure.



Each contour must start from the cut (the free edge). The positive shear flow direction is assumed to be in the contour direction. In the flowing, q_{ij} is used to denote the shear flow between stringer i and j.

Cell 1:



$$q'_{61} = 0$$

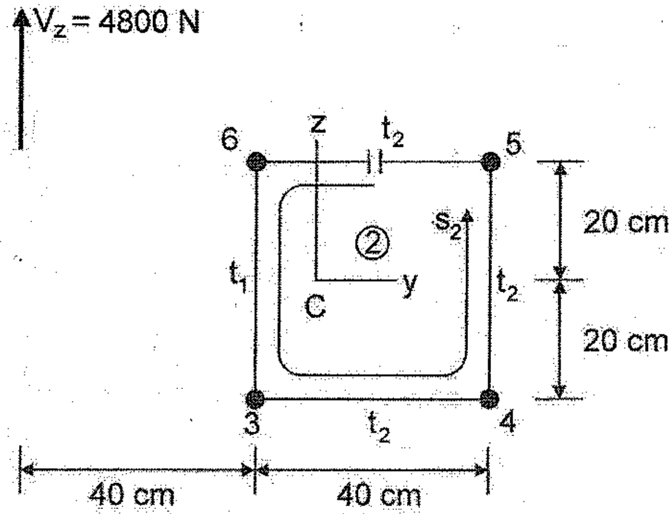
$$Q'_{y,12} = A_1 z_1 = (1cm^2)(20cm) = 20cm^3$$

$$q'_{12} = q'_{61} - \frac{V_z}{I_y} Q'_{y,12} = 0 - \frac{4{,}800N}{4{,}800cm^4} 20cm^3 = -20\frac{N}{cm}$$

$$Q'_{y,23} = A_2 z_2 = (1cm^2)(-20cm) = -20cm^3$$

$$q'_{23} = q'_{12} - \frac{V_z}{I_y} Q'_{y,23} = -20\frac{N}{cm} - \frac{4{,}800N}{4{,}800cm^4}\left(-20cm^3\right) = 0$$

Cell 2:



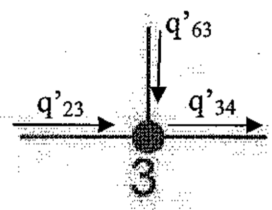
$$q'_{56} = 0$$

$$Q'_{y,63} = A_6 z_6 = (2cm^2)(20cm) = 40cm^3$$

$$q'_{63} = q'_{56} - \frac{V_z}{I_y} Q'_{y,63} = 0 - \frac{4{,}800N}{4{,}800cm^4} 40cm^3 = -40\frac{N}{cm}$$

$$Q'_{y,34} = A_3 z_3 = (2cm^2)(-20cm) = -40cm^3$$

And because,

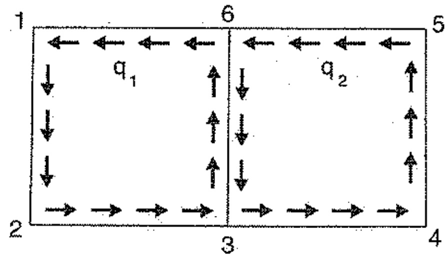


$$q'_{34} = q'_{23} + q'_{63} - \frac{V_z}{I_y} Q'_{y,34} = 0 - 40\frac{N}{cm} - \frac{4{,}800N}{4{,}800cm^4}(-40cm^3) = 0$$

$$Q'_{y,45} = A_4 z_4 = (3cm^2)(20cm) = 60cm^3$$

$$q'_{45} = q'_{34} - \frac{V_z}{I_y} Q'_{y,45} = 0 - \frac{4{,}800N}{4{,}800cm^4} 60cm^3 = -60\frac{N}{cm}$$

The Shear flow are completed by adding the constant shear flows q_1 and q_2 (see the figure) in the individual cells, respectively. The equations needed for determining q_1 and q_2 are obtained from the moment equation and the compatibility equation.



Moment Equation: The in-plane moment produced by V_z about any axis must be equal to the in-plane moment about the same axis resulting from the shear flows. Taking the moment about stringer 1, we have

$$V_z \cdot 0 = 2\overline{A}_1 q_1 + 2\overline{A}_2 q_2 + q'_{23}(40cm)(40cm) + q'_{34}(40cm)(40cm) + q'_{45}(40cm)(80cm) - q'_{63}(40cm)(40cm)$$

where $\overline{A}_1 = \overline{A}_2 = (40cm)(40cm) = 1{,}600cm^2$

Substituting the numerical values of q'_{ij}, we obtain

$$0 = 3{,}200(q_1 + q_2) + 1{,}600(q'_{23} + q'_{34} - q'_{63}) + 3{,}200(q'_{45})$$

$$0 = 3{,}200(q_1 + q_2) + 1{,}600(0 + 0 - (-40)) + 3{,}200(60)$$

$$0 = 3{,}200(q_1 + q_2) + 64{,}000 + 192{,}000$$

$$q_1 + q_2 = \frac{-64{,}000 - 192{,}000}{3{,}200}$$

$$q_1 + q_2 = -80\frac{N}{cm}$$

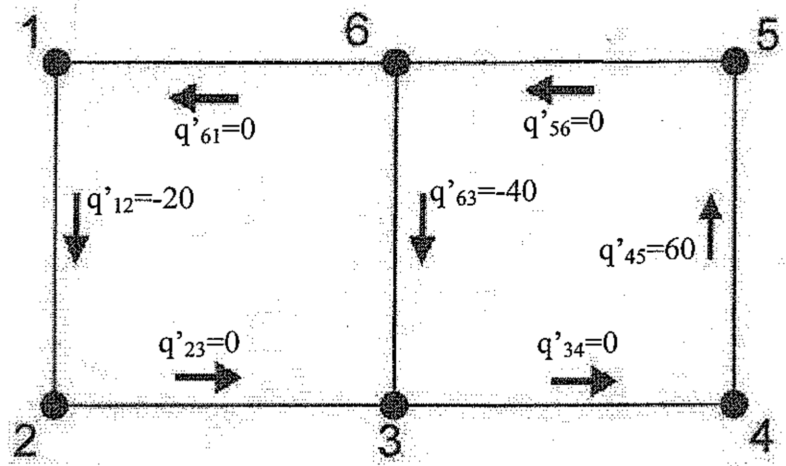
Compatibility Equation: The compatibility condition requires that the twist angle of Cell 1 must be equal to that of Cell 2.

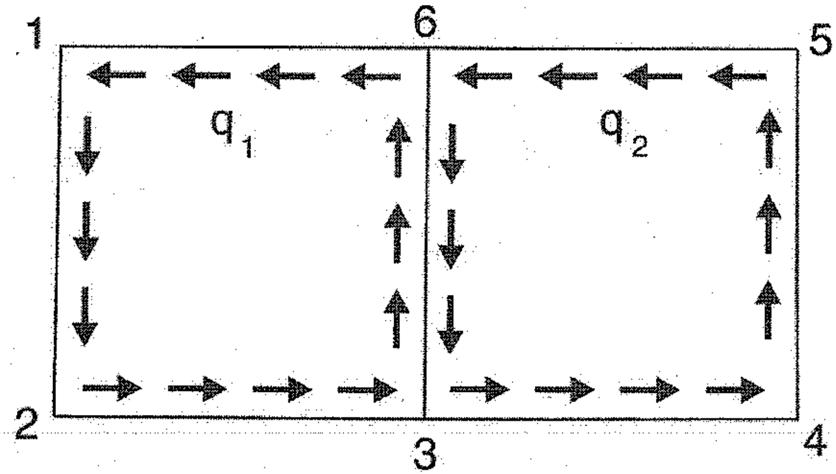
$$\theta = \frac{1}{2G\overline{A}}\oint \frac{q}{t} ds$$

$$\theta_{CELL1} = \theta_{CELL2}$$

$$\frac{1}{2G\overline{A}_1}\left[\frac{q_{61}}{t_1}40+\frac{q_{12}}{t_1}40+\frac{q_{23}}{t_1}40-\frac{q_{63}}{t_1}40\right]=\frac{1}{2G\overline{A}_2}\left[\frac{q_{56}}{t_2}40+\frac{q_{63}}{t_1}40+\frac{q_{34}}{t_2}40+\frac{q_{45}}{t_2}40\right]$$

And,





$$q_{61} = q_{61}^{"} + q_1 = q_1$$
$$q_{12} = q_{12}^{'} + q_1 = q_1 - 20$$
$$q_{23} = q_{23}^{'} + q_1 = q_1$$
$$q_{63} = q_{63}^{'} - q_1 + q_2 = -40 - q_1 + q_2$$
$$q_{34} = q_{34}^{'} + q_2 = q_2$$
$$q_{45} = q_{45}^{'} + q_2 = 60 + q_2$$
$$q_{56} = q_{56}^{'} + q_2 = q_2$$

Substituting back,

$$\frac{40}{2G\bar{A}_1 t_1}\left[q_{61}+q_{12}+q_{23}-q_{63}\right]=\frac{40}{2G\bar{A}_2}\left[\frac{q_{56}}{t_2}+\frac{q_{63}}{t_1}+\frac{q_{34}}{t_2}+\frac{q_{45}}{t_2}\right]$$

$$\left[q_{61}+q_{12}+q_{23}-q_{63}\right]=t_1\left[\frac{q_{56}}{t_2}+\frac{q_{63}}{t_1}+\frac{q_{34}}{t_2}+\frac{q_{45}}{t_2}\right]$$

$$\left[q_{61}+q_{12}+q_{23}-q_{63}\right]=\frac{t_1}{t_2}\left[q_{56}+\frac{t_2}{t_1}q_{63}+q_{34}+q_{45}\right]$$

$$\left[(q_1)+(q_1-20)+(q_1)-(-40-q_1+q_2)\right]=\frac{1}{2}\left[(q_2)+2(-40-q_1+q_2)+(q_2)+(60+q_2)\right]$$

$$2\left[4q_1-q_2+20\right]=\left[(q_2)+(-80-2q_1+2q_2)+(q_2)+(60+q_2)\right]$$

$$\left[8q_1-2q_2+40\right]=\left[5q_2-20-2q_1\right]$$

$$q_1(8+2)+q_2(-2-5)=-60$$

$$10q_1-7q_2=-60$$

Solving these system of equations,

$$\begin{cases} q_1+q_2=-80 \\ 10q_1-7q_2=-60 \end{cases}$$

We get that,

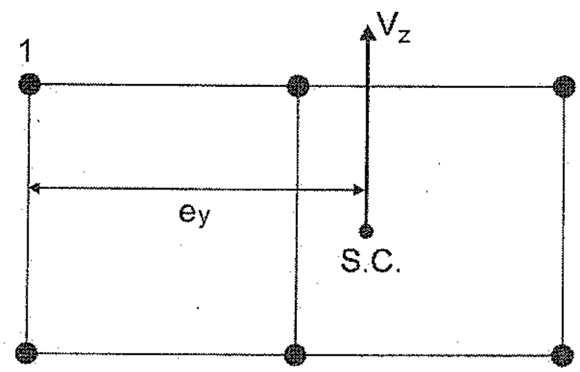
$$q_1+q_2=-80 \quad \Rightarrow \quad q_1=-80-q_2$$

$$10q_1-7q_2=-60 \quad \Rightarrow \quad 10(-80-q_2)-7q_2=-60 \quad \Rightarrow \quad -800-17q_2=-60 \quad \Rightarrow$$

$$\boxed{q_2=-\frac{740}{17}=-43.53\frac{N}{cm}}$$

$$q_1=-80-q_2 \quad \Rightarrow \quad q_1=-80-(-43.53) \quad \Rightarrow \quad \boxed{q_1=-36.47\frac{N}{cm}}$$

Shear Center: To find the shear center, we assume that the applied force passes through the shear center as shown in the figure. The resultant torque of the shear flow and the torque produced by V_z must be equal. Taking the moment about stringer 1, we have,



$$V_z e_y = 2\overline{A}_1 q_1 + 2\overline{A}_2 q_2 + q'_{23}(40cm)(40cm) + q'_{34}(40cm)(40cm) + q'_{45}(40cm)(80cm) - q'_{63}(40cm)(40cm)$$

$$V_z e_y = 3{,}200(q_1 + q_2) + 64{,}000 + 192{,}000$$

$$V_z e_y = 3{,}200(q_1 + q_2) + 256{,}000$$

By definition of shear center, we require that,

$$\theta_1 = 0 = [q_{61} + q_{12} + q_{23} - q_{63}]$$

$$\theta_2 = 0 = \left[q_{56} + \frac{t_2}{t_1} q_{63} + q_{63} + q_{63}\right]$$

These three equations are sufficient to solve for q_1, q_2, and the shear center location e_y. The solutions are

From the first equation,

$$4{,}800 e_y = 3{,}200(q_1 + q_2) + 256{,}000$$

$$q_1 + q_2 = \frac{4{,}800 e_y - 256{,}000}{3{,}200} = 1.5 e_y - 80$$

From the second equation,

$$\left[q_{61} + q_{12} + q_{23} - q_{63}\right] = 0$$

$$4q_1 - q_2 + 20 = 0$$

From the third equation,

$$\left[q_{56} + \frac{t_2}{t_1} q_{63} + q_{63} + q_{63}\right] = 0$$

$$5q_2 - 20 - 2q_1 = 0$$

Solving for q_1 and q_2 we get that,

$$4q_1 - q_2 + 20 = 0 \quad \Rightarrow \quad q_2 = 4q_1 + 20$$

$$5q_2 - 20 - 2q_1 = 0 \quad \Rightarrow \quad 5(4q_1 + 20) - 20 - 2q_1 = 0 \quad \Rightarrow \quad 20q_1 + 100 - 20 - 2q_1 = 0 \quad \Rightarrow$$

$$18q_1 = -80 \quad \Rightarrow \boxed{q_1 = -\frac{80}{18} = -4.44\frac{N}{cm}}$$

$$q_2 = 4q_1 + 20 \quad \Rightarrow \quad q_2 = 4(-4.44) + 20 \quad \Rightarrow \boxed{q_2 = 2.22\frac{N}{cm}}$$

Substituting back,

$$q_1 + q_2 = 1.5e_y - 80$$

$$-4.44 + 2.22 = 1.5e_y - 80$$

$$1.5e_y = -2.22 + 80$$

$$e_y = \frac{77.78}{1.5}$$

$$\boxed{e_y = 51.85cm}$$