MAE3315
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EXAM #2 SOLUTION

<u>Problem 1:</u>

Part a) Centroid

First, take the Aluminum as a reference material.

$$A_{ti}^* = \frac{E_{ti}}{E_{al}} A_{ti} = \frac{100x10^9 Pa}{70x10^9 Pa}(0.12m)(0.008m) = 1.3714x10^{-3} m^2$$

$$A_{al}^* = \frac{E_{al}}{E_{al}} A_{al} = \frac{70x10^9 Pa}{70x10^9 Pa}(0.08m)(0.008m) = 6.40x10^{-4} m^2$$

To calculate the centroid,

$$y_c^* = \frac{\sum A_i^* y_i}{\sum A_i^*} = \frac{(1.3714x10^{-3} m^2)(0.060m) + (6.4x10^{-4} m^2)(0.04m)}{1.3714x10^{-3} m^2 + 6.4x10^{-4} m^2} = 0.0536m$$

$$z_c^* = \frac{\sum A_i^* z_i}{\sum A_i^*} = \frac{(1.3714x10^{-3} m^2)(0.084m) + (6.4x10^{-4} m^2)(0.04m)}{1.3714x10^{-3} m^2 + 6.4x10^{-4} m^2} = 0.0700m$$

Part b) I_y

To calculate the moment of inertia,

$$I_y = \sum I_{yi}^* = \frac{E_{ti}}{E_{al}}\left(I_0 + Ad^2\right)_{ti} + \left(I_0 + Ad^2\right)_{al}$$

$$I_y = \frac{100x10^9 Pa}{70x10^9 Pa}\left(\frac{1}{12}(0.12m)(0.008m)^3 + (0.12m)(0.008m)(0.084m - 0.0700m)^2\right) +$$
$$+\left(\frac{1}{12}(0.008m)(0.08m)^3 + (0.008m)(0.08m)(0.04m - 0.0700m)^2\right) = 1.1934x10^{-6} m^4$$

Problem 2:

Part a) Max Bending Stress due to P

First, the moments are

$$M_y = -PL$$
$$M_z = 0$$

Therefore,

$$\sigma_x = (\overset{0}{\cancel{M_z}}K_y - M_y K_{yz})y + (M_y K_z - \overset{0}{\cancel{M_z}}K_{yz})z$$

$$\sigma_x = (-M_y K_{yz})y + (M_y K_z)z$$

$$\sigma_x = (K_z z - K_{yz} y)M_y$$

Where,

$$K_z = \frac{I_z}{\Delta} \qquad \text{and} \qquad K_{yz} = \frac{I_{yz}}{\Delta}$$

And,

$$\Delta = I_y I_z - I_{yz}^2 = \left(\frac{2}{3}th^3\right)\left(\frac{8}{3}th^3\right) - \left(-th^3\right)^2 = \frac{7}{9}t^2h^6$$

Therefore,

$$K_z = \frac{I_z}{\Delta} = \frac{\frac{8}{3}th^3}{\frac{7}{9}t^2h^6} = \frac{24}{7}\frac{1}{th^3}$$

$$K_{yz} = \frac{I_{yz}}{\Delta} = \frac{-th^3}{\frac{7}{9}t^2h^6} = -\frac{9}{7}\frac{1}{th^3}$$

As a result,

$$\sigma_x = (K_z z - K_{yz} y)M_y$$

$$\sigma_x = \left(\frac{24}{7} \frac{1}{th^3} z + \frac{9}{7} \frac{1}{th^3} y \right) (-PL)$$

$$\sigma_x = \frac{-9PL}{7th^3} y + \frac{-24PL}{7th^3} z$$

There are 2 critical points: The first one, at the most top-right part of the cross-section; the second one, at the most bottom-left part of it. As a result,

$$\sigma_{x(TOP\_RIGHT)} = \frac{-9PL}{7th^3}(h) + \frac{-24PL}{7th^3}(h)$$

$$\sigma_{x(TOP\_RIGHT)} = \frac{-33PL}{7th^2}$$

It makes sense that the top part of the beam is under compression. And the other one,

$$\sigma_{x(BOTTOM-LEFT)} = \frac{-9PL}{7th^3}(-h) + \frac{-24PL}{7th^3}(-h)$$

$$\sigma_{x(BOTTOM-LEFT)} = \frac{33PL}{7th^2}$$

Part b) Neutral Axis

To calculate the neutral axis,

$$\sigma_x = \left(\frac{24}{7} \frac{1}{th^3} z + \frac{9}{7} \frac{1}{th^3} y \right) M_y = 0$$

$$\tan\alpha = -\frac{z}{y} = -\frac{-\frac{9}{7}\frac{1}{th^3}}{\frac{24}{7}\frac{1}{th^3}} = +\frac{3}{8}$$

$$\alpha = \tan^{-1}\left(\frac{3}{8}\right)$$

$\alpha = 0.3588\,\text{rad}$ or $\alpha = 20.55^\circ$ [CLOCKWISE]

Problem 3:

First, the location of the centroid is calculated,

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{(b/2)(bt) + (0)(2bt) + (b/2)(bt)}{bt + 2bt + bt} = \frac{b}{4}$$

$$z_c = 0$$

The moment of inertia is,

$$I_z = \frac{1}{12}\left[(b + t/2)^3(2b + t) - (b - t/2)^3(2b - t)\right] = 8b^3 t + 2bt^3$$

Assuming that $t << b$, and $t^3 \approx 0$

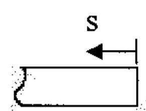
$$I_z = 8b^3 t$$

Because symmetry, the shear flow of each section is,

$$q_s = -\frac{V_y Q_z}{I_z}$$

Where Q_z is, $Q_z = A^* y^*$ for each section.

For Section #1: $0 \le s \le b$

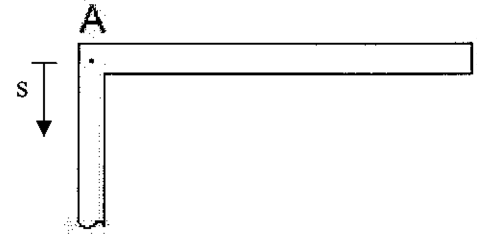


$$Q_1 = st\left[\left(b - \frac{b}{4}\right) - \frac{s}{2}\right] = \frac{3}{4}tbs - \frac{1}{2}ts^2$$

$$q_1 = -\frac{V_y Q_1}{I_z} = -\frac{V_y\left(\frac{3}{4}tbs - \frac{1}{2}ts^2\right)}{8b^3 t} = -\frac{V_y\left(\frac{3}{4}bs - \frac{1}{2}s^2\right)}{8b^3}$$

$$q_1^A = -\frac{V_y\left(\frac{3}{4}b^2 - \frac{1}{2}b^2\right)}{8b^3} = -\frac{V_y}{32b}$$

For Section #2: $0 \le s \le 2b$

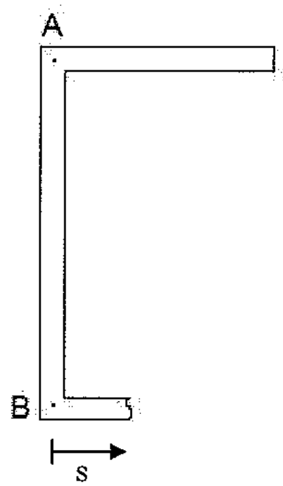


$$Q_2 = st\left[-\frac{b}{4}\right] = -\frac{1}{4}tbs$$

$$q_2 = q_1^A - \frac{V_y Q_2}{I_z} = -\frac{V_y}{32b} - \frac{V_y\left(-\frac{1}{4}tbs\right)}{8b^3 t} = -\frac{V_y}{32b} + \frac{V_y}{32b^2}s = \frac{V_y}{32b^2}(s-b)$$

$$q_2^B = \frac{V_y}{32b^2}(2b-b) = \frac{V_y}{32b}$$

For Section #3: $0 \le s \le b$



$$Q_3 = st\left[\frac{s}{2} - \frac{b}{4}\right] = \frac{1}{2}ts^2 - \frac{1}{4}tbs$$

$$q_3 = q_2^B - \frac{V_y Q_3}{I_z} = \frac{V_y}{32b} - \frac{V_y\left(\frac{1}{2}ts^2 - \frac{1}{4}tbs\right)}{8b^3 t} = \frac{V_y}{32b} - \frac{V_y\left(\frac{1}{2}s^2 - \frac{1}{4}bs\right)}{8b^3}$$

$$q_3^C = \frac{V_y}{32b} - \frac{V_y\left(\frac{1}{2}b^2 - \frac{1}{4}b^2\right)}{8b^3} = \frac{V_y}{32b} - \frac{V_y\left(\frac{1}{4}b^2\right)}{8b^3} = \frac{V_y}{32b} - \frac{V_y}{32b} = 0$$

As expected $q_3^C = 0$!!!!!

Problem 4:

Part a) Centroid

$$A_1 = A_6 = 10in^2 = A \qquad A_2 = A_5 = 2A \qquad A_3 = A_4 = 3A$$

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{2[(20in)A + (10in)2A + (0in)3A]}{2(A + 2A + 3A)} = \frac{40A}{6A} = \frac{20}{3} = 6.6667in$$

$$z_c = 5in \quad \text{(because symmetry)}$$

Part b) Moments of Inertia

$$I_y = \sum_i A_i z_i^2 = 2\left[(10in^2)(5in)^2 + (20in^2)(5in)^2 + (30in^2)(5in)^2\right] = 3{,}000in^4 = 3x10^3 in^4$$

$$I_z = \sum_i A_i y_i^2 = 2\left[(10in^2)\left(20in - \frac{20}{3}in\right)^2 + (20in^2)\left(10in - \frac{20}{3}in\right)^2 + (30in^2)\left(0in - \frac{20}{3}in\right)^2\right] =$$

$$= 20\left[\left(\frac{40}{3}\right)^2 + 2\left(\frac{10}{3}\right)^2 + 3\left(-\frac{20}{3}\right)^2\right] = \frac{20}{9}[1600 + 200 + 1200] = \frac{20}{9}[3000] = \frac{2}{3}x10^4 = 6.667x10^3 in^4$$

Part c) Shear flow with a cut at the web between Stringer 1 and 6.

Once again, because symmetry, the shear flow of each section is reduced to

$$q_s' = -\frac{V_z Q_y'}{I_y}$$

Where Q_y' is, $Q_y' = A^* z^*$ for each section.

For Section #1-2:

$$Q_1' = A_1 z_1^* = (10in^2)(5in) = 50in^3$$

$$q_{12}' = -\frac{(1{,}000lb)(50in^3)}{(3{,}000in^4)} = -\frac{50}{3} lb/in$$

For Section #2-3:

$$Q_2' = A_2 z_2^* = (20in^2)(5in) = 100in^3$$

$$q_{23}' = q_{12}' - \frac{(1{,}000lb)(100in^3)}{(3{,}000in^4)} = -\frac{50}{3} - \frac{100}{3} = -50lb/in$$

For Section #3-4:

$$Q_3' = A_3 z_3^* = (30in^2)(5in) = 150in^3$$

$$q_{34}' = q_{23}' - \frac{(1{,}000lb)(150in^3)}{(3{,}000in^4)} = -50 - 50 = -100lb/in$$

For Section #4-5:

$$q_{45}' = q_{23}' = -50lb/in$$

For Section #5-6:

$$q_{56}' = q_{12}' = -\frac{50}{3}lb/in$$

Part d) Shear flow of each segment for the closed section.

Taking summation of moments at "3",

$$\left(\frac{20}{3}\right)V_z = 2\overline{A}q_0 + (10)q_{45}'(10) + (10in)q_{56}'(10in)$$

$$\left(\frac{20}{3}\right)(1000) = 2(20)(10)q_0 + (10)(-50)(10) + (10)\left(-\frac{50}{3}\right)(10)$$

$$q_0 = \frac{100}{3} = 33.3333lb/in$$

Finally,

$$q_{12} = q_0 + q_{12}' = \frac{100}{3} - \frac{50}{3} = \frac{50}{3}lb/in$$

$$q_{23} = q_0 + q_{23}' = \frac{100}{3} - 50 = -\frac{50}{3} lb/in$$

$$q_{34} = q_0 + q_{34}' = \frac{100}{3} - 100 = -\frac{200}{3} lb/in$$

$$q_{45} = q_0 + q_{45}' = \frac{100}{3} - 50 = -\frac{50}{3} lb/in$$

$$q_{56} = q_0 + q_{56}' = \frac{100}{3} - \frac{50}{3} = \frac{50}{3} lb/in$$

$$q_{61} = q_0 + q_{61}' = \frac{100}{3} + 0 = \frac{100}{3} lb/in$$

Part e) Shear center

Because of symmetry about "z" axis, the shear center lies on this axis. Therefore, we only need to find y_{sc}.

Taking again summation of moments at "3" but this time V_z is not applied on the centroid but through the shear center (assumed to be at distance e_y from "1")

$$(e_y)V_z = 2\overline{A}q_0 + (10in)q_{45}'(10in) + (10in)q_{56}'(10in)$$

$$(e_y)(1000) = 2(20)(10)q_0 + (10)(-50)(10) + (10)\left(-\frac{50}{3}\right)(10)$$

$$2(20)(10)q_0 = 1000e_y + 5000 + \frac{5000}{3}$$

$$q_0 = \frac{1000}{400}e_y + \frac{5000}{400} + \frac{5000}{3(400)}$$

$$q_0 = \frac{50}{3} + \frac{5}{2}e_y$$

Recalculating,

$$q_{12} = q_0 + q_{12}' = \frac{50}{3} + \frac{5}{2}e_y - \frac{50}{3} = \frac{5}{2}e_y$$

$$q_{23} = q_0 + q_{23}' = \frac{50}{3} + \frac{5}{2}e_y - 50 = -\frac{100}{3} + \frac{5}{2}e_y$$

$$q_{34} = q_0 + q_{34}' = \frac{50}{3} + \frac{5}{2}e_y - 100 = -\frac{250}{3} + \frac{5}{2}e_y$$

$$q_{45} = q_0 + q_{45}' = \frac{50}{3} + \frac{5}{2}e_y - 50 = -\frac{100}{3} + \frac{5}{2}e_y$$

$$q_{56} = q_0 + q_{56}' = \frac{50}{3} + \frac{5}{2}e_y - \frac{50}{3} = \frac{5}{2}e_y$$

$$q_{61} = q_0 + q_{61}' = \frac{50}{3} + \frac{5}{2}e_y + 0 = \frac{50}{3} + \frac{5}{2}e_y$$

Finally, since V_z passes through the shear center, the twist angle is equal to zero.

$$\theta = \frac{1}{2G\overline{A}} \oint q \frac{ds}{t} = 0 \qquad \Rightarrow \qquad \sum q_i$$

$$q_{12} + q_{23} + q_{34} + q_{45} + q_{56} + q_{61} = 0$$

$$\left(\frac{5}{2}e_y\right) + \left(-\frac{100}{3} + \frac{5}{2}e_y\right) + \left(-\frac{250}{3} + \frac{5}{2}e_y\right) + \left(-\frac{100}{3} + \frac{5}{2}e_y\right) + \left(\frac{5}{2}e_y\right) + \left(\frac{50}{3} + \frac{5}{2}e_y\right) = 0$$

$$15e_y - \frac{400}{3} = 0$$

$$e_y = \frac{400}{45} = 8.889in$$

Finally,

$$y_{SC} = \frac{400}{45} - \frac{20}{3} = \frac{20}{9} = 2.222in$$

Take Home Problem:

Part a) Centroid

$$A_1 = A_4 = 1in^2 = A \qquad A_2 = A_3 = 2A$$

$$y_c = \frac{\sum y_i A_i}{\sum A_i} = \frac{(10in)A + (10in)A}{A + 2A + 2A + A} = \frac{20}{6} = \frac{10}{3} = 3.3333in$$

$$z_c = 2.5in \text{ (because symmetry)}$$

Part b) Moments of Inertia

$$I_y = \sum_i A_i z_i^2 = A(5-2.5)^2 + 2A(5-2.5)^2 + 2A(0-2.5)^2 + A(0-2.5in)^2 = \frac{300}{8} = 37.5in^4$$

$$I_z = \sum_i A_i y_i^2 = A\left(10 - \frac{10}{3}\right)^2 + 2A\left(0 - \frac{10}{3}\right)^2 + 2A\left(0 - \frac{10}{3}\right)^2 +$$
$$+ A\left(10 - \frac{10}{3}\right)^2 = \frac{400}{3} = 133.3333in^4$$

Because symmetry, I_{yz} must be "0", let's check,

$$I_{yz} = \sum_i A_i y_i z_i = A\left(\frac{20}{3}\right)(2.5in) + 2A\left(-\frac{10}{3}\right)(2.5in) +$$
$$+ 2A\left(-\frac{10}{3}\right)(-2.5in) + A\left(\frac{20}{3}\right)(-2.5in) = 0in^4$$

Check it.!!!!

Part c) Shear flow with a cut at the web between Stringer 1 and 4.

Once again, because symmetry, the shear flow of each section is reduced to

$$q_s' = -\frac{V_y Q_z'}{I_z}$$

Where Q_z' is, $Q_z' = A^* y^*$ for each section.

For Section #1:

$$Q_1' = A_1 y_1^* = \left(1in^2\right)\left(\frac{20}{3}in\right) = \frac{20}{3}in^3$$

$$q_{12}' = -\frac{(500lb)\left(\frac{20}{3}in^3\right)}{\left(\frac{400}{3}in^4\right)} = -25lb/in$$

For Section #2:

$$Q_2' = A_2 y_2^* = \left(2in^2\right)\left(-\frac{10}{3}in\right) = -\frac{20}{3}in^3$$

$$q_{23}' = q_{12}' - \frac{V_y Q_2'}{I_z} = -25\frac{lb}{in} - \frac{(500lb)\left(-\frac{20}{3}in^3\right)}{\left(\frac{400}{3}in^4\right)} = -25 + 25 = 0lb/in$$

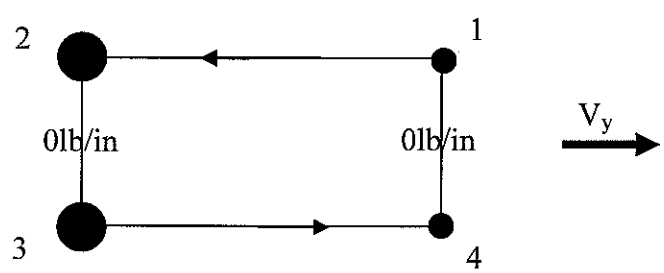
For Section #3:

$$q_{34}' = -q_{12}' = 25lb/in$$

For Section #4:

$$q_{41}' = 0lb/in$$

Part d) Shear flow for each segment.



Taking summation of moments at the centroid,

$$V_y(0) = 2\overline{A}q_0 + (10)q_{12}'(2.5) + (10)q_{34}'(2.5)$$

$$0 = 2(5)(10)q_0 + (10)(-25)(2.5) + (10)(25)(2.5)$$

$$q_0 = 0lb/in$$

As a result,

$$q_{12} = q_0 + q_{12}^{'} = -25lb/in$$

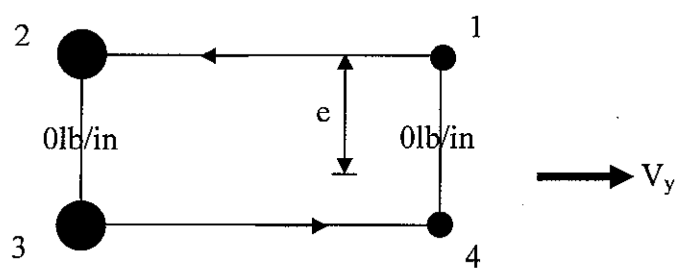
$$q_{23} = q_0 + q_{23}^{'} = 0lb/in$$

$$q_{34} = q_0 + q_{34}^{'} = 25lb/in$$

$$q_{41} = q_0 + q_{41}^{'} = 0lb/in$$

Part e) Shear center.

z_{SC} will be located at $z = 0$ because of symmetric cross-section,



Prove: Taking summation of moments at 1,

$$V_y(e) = 2\overline{A}q_0 + (5)q_{23}^{'}(10) + (10)q_{34}^{'}(5)$$

$$500(e) = 2(5)(10)q_0 + (10)(25)(5)$$

$$q_0 = 5e - 12.5$$

Therefore,

$$q_{12} = q_0 + q_{12}^{'} = 5e - 12.5 - 25 = 5e - 37.5$$

$$q_{23} = q_0 + q_{23}^{'} = 5e - 12.5 + 0 = 5e - 12.5$$

$$q_{34} = q_0 + q_{34}^{'} = 5e - 12.5 + 25 = 5e + 12.5$$

$$q_{41} = q_0 + q'_{41} = 5e - 12.5 + 0 = 5e - 12.5$$

Finally,

$$\theta = \frac{1}{2G\bar{A}} \oint q \frac{ds}{t} = 0$$

$$q_{12} \frac{10}{t} + q_{23} \frac{5}{t} + q_{34} \frac{10}{t} + q_{41} \frac{5}{t} = 0$$

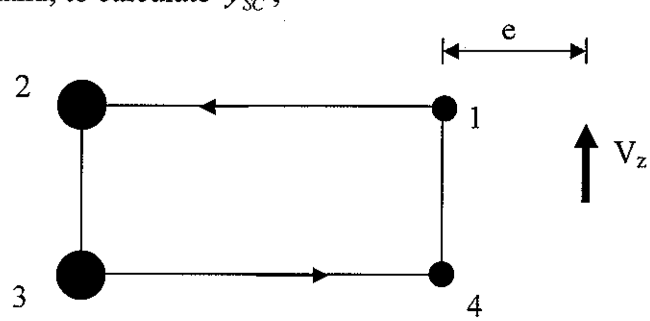
$$(5e - 37.5)10 + (5e - 12.5)5 + (5e + 12.5)10 + (5e - 12.5)5 = 0$$

$$(50e - 375) + (25e - 62.5) + (50e + 125) + (25e - 62.5) = 0$$

$$150e - 375 = 0$$

$$e = \frac{375}{150} = \frac{5}{2} = 2.5in$$

On the other hand, to calculate y_{SC},



In this case, $q = -\frac{V_z Q_y}{I_y}$,

$$q'_{12} = -\frac{V_z(1)(2.5)}{300/8} = -\frac{20}{300} V_z = -\frac{2}{30} V_z$$

$$q'_{23} = q'_{12} - \frac{V_z(2)(2.5)}{300/8} = -\frac{2}{30} V_z - \frac{4}{30} V_z = -\frac{1}{5} V_z$$

$$q'_{34} = q'_{12} = -\frac{2}{30} V_z$$

Taking moments at 1,

$$V_z(e) = 2\bar{A}q_0 + (5)q'_{23}(10) + (10)q'_{34}(5)$$

$$V_z(e) = 2(5)(10)q_0 + (5)\left(-\frac{1}{5}V_z\right)(10) + (10)\left(-\frac{2}{30}V_z\right)(5)$$

$$2(5)(10)q_0 = V_z(e) + \frac{260}{3}V_z$$

$$q_0 = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z$$

As usually,

$$q_{12} = q_0 + q'_{12} = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z - \frac{2}{30}V_z = \left[\frac{1}{100}e + \frac{1}{15}\right]V_z$$

$$q_{23} = q_0 + q'_{23} = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z - \frac{1}{5}V_z = \left[\frac{1}{100}e - \frac{1}{15}\right]V_z$$

$$q_{34} = q_0 + q'_{34} = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z - \frac{2}{30}V_z = \left[\frac{1}{100}e + \frac{1}{15}\right]V_z$$

$$q_{41} = q_0 + q'_{41} = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z + 0 = \left[\frac{1}{100}e + \frac{2}{15}\right]V_z$$

Because the force in "z" is assumed to pass through the shear center, then $\theta = 0$,

$$\theta = \frac{1}{2G\bar{A}}\oint q\frac{ds}{t} = 0$$

$$\left[\frac{1}{100}e + \frac{1}{15}\right]V_z\frac{10}{t} + \left[\frac{1}{100}e - \frac{1}{15}\right]V_z\frac{5}{t} + \left[\frac{1}{100}e + \frac{1}{15}\right]V_z\frac{10}{t} + \left[\frac{1}{100}e + \frac{2}{15}\right]V_z\frac{5}{t} = 0$$

As you can see V_z cancels out,

$$\left[\frac{1}{100}e + \frac{1}{15}\right]10 + \left[\frac{1}{100}e - \frac{1}{15}\right]5 + \left[\frac{1}{100}e + \frac{1}{15}\right]10 + \left[\frac{1}{100}e + \frac{2}{15}\right]5 = 0$$

$$\left[\frac{1}{10}e+\frac{2}{3}\right]+\left[\frac{1}{20}e-\frac{1}{3}\right]+\left[\frac{1}{10}e+\frac{2}{3}\right]+\left[\frac{1}{20}e+\frac{2}{3}\right]=0$$

$$\frac{3}{10}e+\frac{5}{3}=0$$

$$e=-\frac{50}{9}=-5.556in$$

Finally,

$$y_{SC}=(10-3.333)-5.556=1.111in$$

Part f) Bending stress due to $M_y = 500Kips-in$

$$\sigma_x=(\cancelto{0}{M_zK_y}-\cancelto{0}{M_yK_{yz}})y+(M_yK_z-\cancelto{0}{M_zK_{yz}})z$$

$$\sigma_x=(M_yK_z)z$$

Where,

$$K_z=\frac{I_z}{I_yI_z-\cancelto{0}{I_{yz}^2}}=\frac{1}{I_y}=\frac{8}{300}$$

As a result,

$$\sigma_x=(500x10^3lb-in)\left(\frac{8}{300}\frac{1}{in^4}\right)z$$

$$\sigma_x=\frac{4x10^4}{3}z$$

Evaluating the bending stress at 1 and 2, we get that,

$$\sigma_x\Big|_{\text{at 1 \& 2 (z=2.5)}}=\frac{4x10^4}{3}\left(\frac{5}{2}\right)=\frac{1}{3}x10^5=33.3ksi$$

Now evaluating the bending stress again but this time at 3 and 4,

$$\sigma_x\Big|_{\text{at 3 \& 4 (z=-2.5)}}=\frac{4x10^4}{3}\left(-\frac{5}{2}\right)=-\frac{1}{3}x10^5=-33.3ksi$$