Physics 1444
General Technical Physics

Term
Fall 2005

HW#1 Solutions

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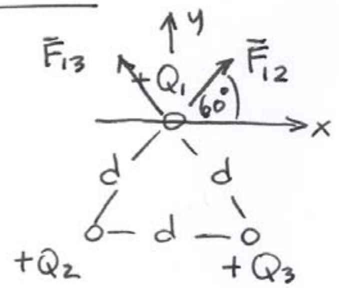
Electricity, Magnetism & Light
Chapter 21 - Homework #1

21.11/

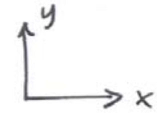
Prob #11



$$\bar{F}_{12} = \left(\frac{1}{4\pi\epsilon_0}\ \frac{Q_1 Q_2}{d^2}\right)\hat{e}_{21}$$

$$\bar{F}_{13} = \left(\frac{1}{4\pi\epsilon_0}\ \frac{Q_1 Q_3}{d^2}\right)\hat{e}_{31}$$

Solve for x, y components



Resolution of vectors into components:

$$\hat{e}_{21} = \cos 60°\,\hat{e}_x + \sin 60°\,\hat{e}_y$$

$$\hat{e}_{31} = -\cos 60°\,\hat{e}_x + \sin 60°\,\hat{e}_y$$

$$\bar{F}_{12} = \left(\frac{1}{4\pi\epsilon_0}\ \frac{Q_1 Q_2}{d^2}\right)\left(\cos 60°\,\hat{e}_x + \sin 60°\,\hat{e}_y\right)$$

$Q_2 = Q_3$

$$\bar{F}_{13} = \left(\frac{1}{4\pi\epsilon_0}\ \frac{Q_1 Q_2}{d^2}\right)\left(-\cos 60°\,\hat{e}_x + \sin 60°\,\hat{e}_y\right)$$

$$\bar{F}_{tot} = \bar{F}_{12} + \bar{F}_{13}$$

$$= \frac{2}{4\pi\epsilon_0}\left(\frac{Q_1 Q_2}{d^2}\right)\sin 60°\,\hat{e}_y$$

The remaining forces will have the same magnitude, but different direction.

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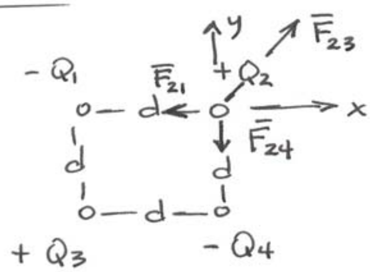
Prob. #13



One force calculation to illustrate the method.

$$\bar{F}_{23} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_3}{(\sqrt{2}d)^2}\right) \hat{e}_{32}$$

$$\bar{F}_{21} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_1}{d^2}\right) \hat{e}_{12} \qquad \hat{e}_{12} = \hat{e}_x$$

$$\bar{F}_{24} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_4}{d^2}\right) \hat{e}_{42} \qquad e_{42} = +\hat{e}_y$$

Next resolve $\bar{F}_{23}$ into x, y components

$$\bar{F}_{23} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_3}{(\sqrt{2}d)^2}\right)\left(\cos 45^\circ\, \hat{e}_x + \sin 45^\circ\, \hat{e}_y\right)$$

Add all forces to get the total:

$$\bar{F}_{tot(2)} = \bar{F}_{23} + \bar{F}_{21} + \bar{F}_{24}$$

$$= \left[\frac{1}{4\pi\epsilon_0}\left(\frac{Q_2 Q_3}{(\sqrt{2}d)^2} \cos 45^\circ + \frac{Q_2 Q_1}{d^2}\right)\right] \hat{e}_x$$

$$+ \left[\frac{1}{4\pi\epsilon_0}\left(\frac{Q_2 Q_3}{(\sqrt{2}d)^2} \sin 45^\circ - \frac{Q_2 Q_4}{d^2}\right)\right] \hat{e}_y$$

Note!

The remaining forces are done in a similar fashion.

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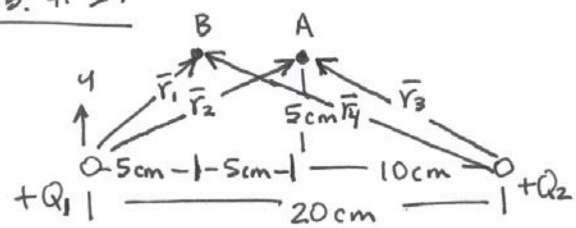
Prob. #37



Do some geometry first

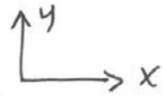
$r_1 = \sqrt{5^2 + 5^2}$ cm

$r_2 = \sqrt{5^2 + 10^2}$ cm

$r_3 = \sqrt{5^2 + 10^2}$ cm

$r_4 = \sqrt{5^2 + 15^2}$ cm

Put the origin of a coordinate system at Q_1



Electric field @ B:

$$\bar{E}_B = \bar{E}_{BQ_1} + \bar{E}_{BQ_2}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\frac{Q_1}{|\bar{r}_1|^2}\right)\hat{r}_1 + \left(\frac{1}{4\pi\epsilon_0}\frac{Q_2}{|\bar{r}_4|^2}\right)\hat{r}_4$$

$\hat{r}_1 = \frac{\bar{r}_1}{|\bar{r}_1|}$ $\quad \hat{r}_4 = \frac{\bar{r}_4}{|\bar{r}_4|}$ unit vectors in directions of $\hat{r}_1, \hat{r}_4$

Resolve $\hat{r}_1$ and $\hat{r}_4$ into components in $\hat{e}_x$ and $\hat{e}_y$:

$$\hat{r}_1 = \left(\tfrac{1}{\sqrt{2}}\right)\hat{e}_x + \left(\tfrac{1}{\sqrt{2}}\right)\hat{e}_y$$

$$\hat{r}_4 = -\left(\tfrac{15}{r_4}\right)e_x + \left(\tfrac{5}{r_4}\right)\hat{e}_y$$

$$\bar{E}_B = \left[\frac{1}{4\pi\epsilon_0}\left(\left(\frac{Q_1}{|\bar{r}_1|^2}\right)\frac{1}{\sqrt{2}} - \left(\frac{Q_2}{|\bar{r}_4|^2}\right)\left(\frac{15}{r_4}\right)\right)\right]\hat{e}_x$$
$$+ \left[\frac{1}{4\pi\epsilon_0}\left(\left(\frac{Q_1}{|\bar{r}_1|^2}\right)\frac{1}{\sqrt{2}} + \left(\frac{Q_2}{|\bar{r}_4|^2}\right)\left(\frac{5}{r_4}\right)\right)\right]\hat{e}_y$$

Force at A is found in a similar way.

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Prob. #42

This problem is a superposition problem. We find the field due to each ring separately and add the results. From Ex. 21-9 in the book, the field for a ring at the origin is

$$\bar{E}_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\,\hat{e}_x \qquad \bar{E}_x = 0 \quad @\ x=0$$

For a ring displaced by $\pm \ell/2$ from the origin we have

$$\bar{E}_{+\ell/2} = \frac{1}{4\pi\epsilon_0}\frac{Q(x-\ell/2)}{[(x-\ell/2)^2+R^2]^{3/2}}\,e_x \qquad \bar{E}_{+\ell/2} = 0\ @\ x = \ell/2$$

$$\bar{E}_{-\ell/2} = \frac{1}{4\pi\epsilon_0}\frac{Q(x+\ell/2)}{[(x+\ell/2)^2+R^2]^{3/2}}\,\hat{e}_x \qquad \bar{E}_{-\ell/2} = 0\ @\ x = -\ell/2$$

$$\bar{E}_{tot} = \bar{E}_{+\ell/2} + \bar{E}_{-\ell/2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{(x-\ell/2)}{[(x-\ell/2)^2+R^2]^{3/2}} + \frac{(x+\ell/2)}{[(x+\ell/2)^2+R^2]^{3/2}}\right]\hat{e}_x$$

$$= 0 \quad @ \quad x = 0$$

The field is zero @ the origin $x=0$ as expected.

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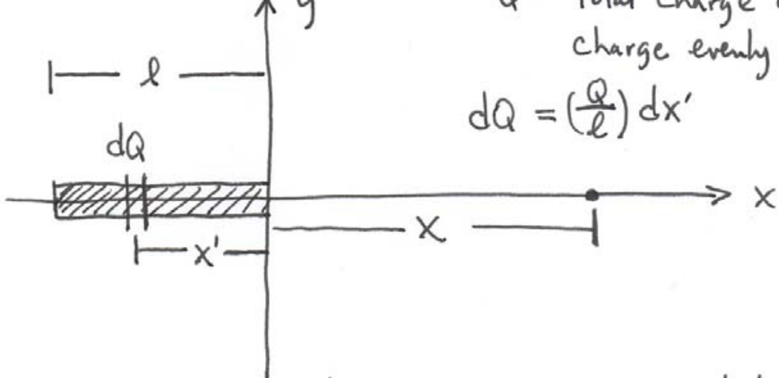
Prob # 48

This problem requires some integration, but the integral is simple. Set up like so.



$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{(x'+x)^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{l}\right)\frac{dx'}{(x'+x)^2}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{l}\right)\int_0^l \frac{dx'}{(x'+x)^2}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{l}\right)\left[\frac{-1}{(x'+x)}\right]_0^l$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{l}\right)\left[\frac{-1}{(l+x)} + \frac{1}{x}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{l}\right)\frac{l}{x(x+l)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(x+l)}$$

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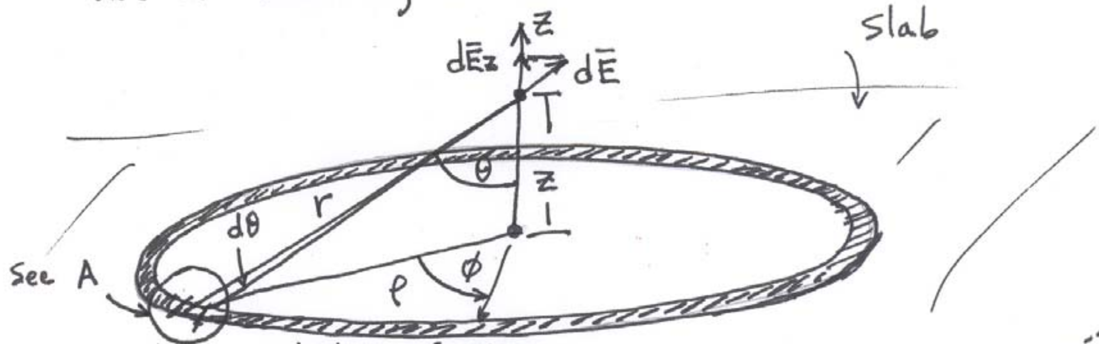
Prob. #53

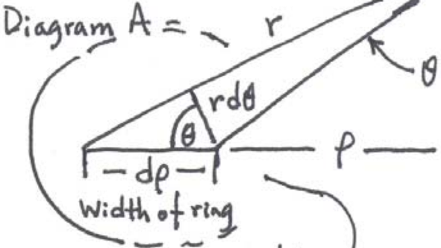
Use an annular ring. It's easier.



$dQ' = \sigma \rho \, d\rho \, d\phi$ inf. charge

$dQ = 2\pi \sigma \rho \, d\rho$ inf. ring

Diagram A =



Solution

$$dE_z = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma\rho \, d\rho \cos\theta}{r^2} \qquad \rho = r\sin\theta \quad d\rho = \frac{r\,d\theta}{\cos\theta}$$

$$= \frac{\sigma}{2\epsilon_0} \frac{r\sin\theta \, \frac{r\,d\theta}{\cos\theta} \cos\theta}{r^2} = \frac{\sigma}{2\epsilon_0} \sin\theta \, d\theta$$

$$E_z = \frac{\sigma}{2\epsilon_0} \int_0^{\pi/2} \sin\theta \, d\theta = \frac{\sigma}{2\epsilon_0} \Big[-\cos\theta \Big]_0^{\pi/2} = \frac{\sigma}{2\epsilon_0}$$

The electric field has the same magnitude everywhere above the infinite slab!