

Series 30 - Inductance

Solutions to Homework #10

Prob. #3

The magnetic field inside the outer solenoid is

$$B = \mu_0 n_1 I_1$$

The magnetic flux linked with the inner solenoid is

$$\Phi_{21} = \mu_0 n_1 I_1 A_2$$

The mutual inductance with the N_2 turns of the inner solenoid is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = n_2 l \mu_0 n_1 A_2$$

Per unit length it's $M_{21}/l = \mu_0 n_1 n_2 \pi r_2^2$

Prob #5

$$\Phi_B = \int \vec{B} \cdot d\vec{s} = \int_{l_1}^{l_2} \frac{\mu_0 I}{2\pi r} w dr = \frac{\mu_0 I w}{2\pi} \ln\left(\frac{l_2}{l_1}\right)$$

The mutual inductance is

$$M = \frac{\Phi_B}{I} = \frac{\mu_0 w}{2\pi} \ln\left(\frac{l_2}{l_1}\right)$$

Prob # 16

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B \cdot 2\pi r = \mu_0 NI \quad \text{gives} \quad B = \frac{\mu_0 NI}{2\pi r}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{s} = \int_{r_1}^{r_2} \frac{\mu_0 NI h}{2\pi r} dr = \frac{\mu_0 NI h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

The flux through the entire toroidal winding is N times this,

so the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

Prob # 23

the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

the energy density of the field is

$$u = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

$$\frac{U}{\ell} = \int \frac{u}{\ell} dv = \int_{r_1}^{r_2} \frac{\mu_0 I^2}{8\pi^2 r^2 \ell} 2\pi r \ell dr = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{r_2}{r_1}\right)$$