mae 3309 – 002 THERMAL ENGINEERING
Solutions to HW # 2

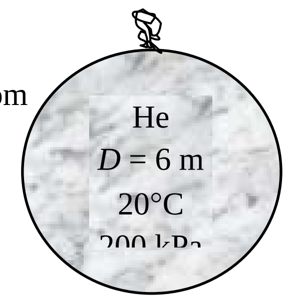
4-77 A balloon is filled with helium gas. The mole number and the mass of helium in the balloon are to be determined.

Assumptions At specified conditions, helium behaves as an ideal gas.

Properties The universal gas constant is R_u = 8.314 kPa.m^3/kmol.K. The molar mass of helium is 4.0 kg/kmol (Table A-1).

Analysis The volume of the sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3\text{ m})^3 = 113.1\text{ m}^3$$



Assuming ideal gas behavior, the mole numbers of He is determined from

$$N = \frac{PV}{R_u T} = \frac{(200\text{ kPa})(113.1\text{ m}^3)}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(293\text{ K})} = \mathbf{9.28\ \ kmol}$$

Then the mass of He can be determined from

$$m = NM = (9.28\text{ kmol})(4.0\text{ kg/kmol}) = \mathbf{37.15\ \ kg}$$

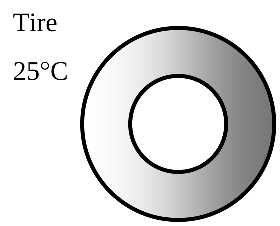
4-79 An automobile tire is inflated with air. The pressure rise of air in the tire when the tire is heated and the amount of air that must be bled off to reduce the temperature to the original value are to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1).

Analysis Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{\text{atm}} = 210 + 100 = 310\text{kPa}$$



Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire can be determined from

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1}P_1 = \frac{323\text{ K}}{298\text{ K}}(310\text{ kPa}) = 336\text{ kPa}$$

Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26\ kPa}$$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(310\text{ kPa})(0.025\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{ K})} = 0.0906\text{ kg}$$

$$m_2 = \frac{P_1 V}{RT_2} = \frac{(310\text{ kPa})(0.025\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\text{ K})} = 0.0836\text{ kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070\ kg}$$

4-88 The specific volume of steam is to be determined using the ideal gas relation, the compressibility chart, and the steam tables. The errors involved in the first two approaches are also to be determined.

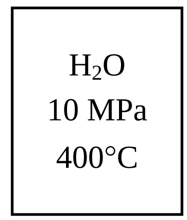
Properties The gas constant, the critical pressure, and the critical temperature of water are, from Table A-1,

$$R = 0.4615 \text{ kPa·m}^3\text{/kg·K}, \qquad T_{cr} = 647.1 \text{ K}, \qquad P_{cr} = 22.06 \text{ MPa}$$

Analysis (*a*) From the ideal gas equation of state,

$$v = \frac{RT}{P} = \frac{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(673 \text{ K})}{(10{,}000 \text{ kPa})} = \mathbf{0.03106 \ \ m^3/kg \ \ (17.6\% \ error)}$$

(*b*) From the compressibility chart (Fig. A-15),



$$\left.\begin{aligned} P_R &= \frac{P}{P_{cr}} = \frac{10 \text{ MPa}}{22.06 \text{ MPa}} = 0.453 \\ T_R &= \frac{T}{T_{cr}} = \frac{673 \text{ K}}{647.1 \text{ K}} = 1.04 \end{aligned}\right\} Z = 0.84$$

Thus,

$$v = Z v_{\text{ideal}} = (0.84)(0.03106 \text{ m}^3/\text{kg}) = \mathbf{0.02609 \ \ m^3/kg \ \ \ (1.2\% \ error)}$$

(*c*) From the superheated steam table (Table A-6),

$$\left.\begin{aligned} P &= 10 \text{ MPa} \\ T &= 400°\text{C} \end{aligned}\right\} v = \mathbf{0.02644 \ \ m^3/kg}$$

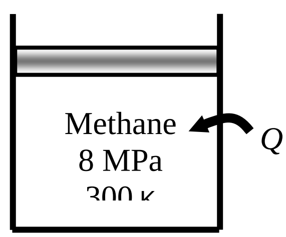
-97 Methane is heated at constant pressure. The final temperature is to be determined using ideal gas equation and the compressibility charts.

Properties The gas constant, the critical pressure, and the critical temperature of methane are, from Table A-1,

$$R = 0.5182 \text{ kPa·m}^3\text{/kg·K}, \qquad T_{cr} = 191.1 \text{ K}, \qquad P_{cr} = 4.64 \text{ MPa}$$

Analysis From the ideal gas equation,

$$T_2 = T_1 \frac{v_2}{v_1} = (300 \text{ K})(1.5) = \mathbf{450 \ K}$$



From the compressibility chart at the initial state (Fig. A-15),

$$\left.\begin{aligned} T_{R1} &= \frac{T_1}{T_{\text{cr}}} = \frac{300 \text{ K}}{191.1 \text{ K}} = 1.570 \\ P_{R1} &= \frac{P_1}{P_{\text{cr}}} = \frac{8 \text{ MPa}}{4.64 \text{ MPa}} = 1.724 \end{aligned}\right\} Z_1 = 0.88, \ v_{R1} = 0.80$$

At the final state,

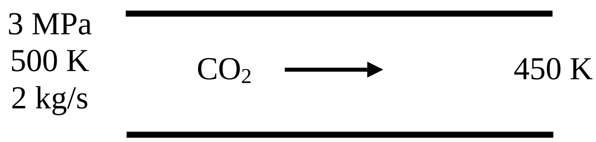
$$\left.\begin{aligned} P_{R2} &= P_{R1} = 1.724 \\ v_{R2} &= 1.5 v_{R1} = 1.5(0.80) = 1.2 \end{aligned}\right\} Z_2 = 0.975$$

Thus,

$$T_2 = \frac{P_2 v_2}{Z_2 R} = \frac{P_2}{Z_2} \frac{v_{R2} T_{\text{cr}}}{P_{\text{cr}}} = \frac{8000 \text{ kPa}}{0.975} \frac{(1.2)(191.1 \text{ K})}{4640 \text{ kPa}} = \mathbf{406 \ K}$$

Of these two results, the accuracy of the second result is limited by the accuracy with which the charts may be read. Accepting the error associated with reading charts, the second temperature is the more accurate.

4-99 CO_2 gas flows through a pipe. The volume flow rate and the density at the inlet and the volume flow rate at the exit of the pipe are to be determined.



Properties The gas constant, the critical pressure, and the critical temperature of CO_2 are (Table A-1)

$$R = 0.1889 \text{ kPa·m}^3\text{/kg·K}, \qquad T_{cr} = 304.2 \text{ K}, \qquad P_{cr} = 7.39 \text{ MPa}$$

Analysis (*a*) From the ideal gas equation of state,

$$\dot{V}_1 = \frac{\dot{m}RT_1}{P_1} = \frac{(2 \text{ kg/s})(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500 \text{ K})}{(3000 \text{ kPa})} = \mathbf{0.06297\ m^3/kg\quad (2.1\%\ error)}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(3000 \text{ kPa})}{(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500 \text{ K})} = \mathbf{31.76\ kg/m^3\quad (2.1\%\ error)}$$

$$\dot{V}_2 = \frac{\dot{m}RT_2}{P_2} = \frac{(2 \text{ kg/s})(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(450 \text{ K})}{(3000 \text{ kPa})} = \mathbf{0.05667\ m^3/kg\quad (3.6\%\ error)}$$

(*b*) From the compressibility chart (EES function for compressibility factor is used)

$$\left.\begin{aligned} P_R &= \frac{P_1}{P_{cr}} = \frac{3 \text{ MPa}}{7.39 \text{ MPa}} = 0.407 \\ T_{R,1} &= \frac{T_1}{T_{cr}} = \frac{500 \text{ K}}{304.2 \text{ K}} = 1.64 \end{aligned}\right\} Z_1 = 0.9791$$

$$\left.\begin{aligned} P_R &= \frac{P_2}{P_{cr}} = \frac{3 \text{ MPa}}{7.39 \text{ MPa}} = 0.407 \\ T_{R,2} &= \frac{T_2}{T_{cr}} = \frac{450 \text{ K}}{304.2 \text{ K}} = 1.48 \end{aligned}\right\} Z_2 = 0.9656$$

Thus,

$$\dot{V}_1 = \frac{Z_1\dot{m}RT_1}{P_1} = \frac{(0.9791)(2 \text{ kg/s})(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500 \text{ K})}{(3000 \text{ kPa})} = \mathbf{0.06165\ m^3/kg}$$

$$\rho_1 = \frac{P_1}{Z_1RT_1} = \frac{(3000 \text{ kPa})}{(0.9791)(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500 \text{ K})} = \mathbf{32.44\ kg/m^3}$$

$$\dot{V}_2 = \frac{Z_2\dot{m}RT_2}{P_2} = \frac{(0.9656)(2 \text{ kg/s})(0.1889 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(450 \text{ K})}{(3000 \text{ kPa})} = \mathbf{0.05472\ m^3/kg}$$