Math 1426

Spring 2004 **Final exam – Version A**

The square brackets following an exam question number refer to a section/problem number in the text or a lab worksheet. Problems numbers preceded by the symbol ~ are modeled on that problem from the text or lab, but not identical to it. Problems numbers without the symbol are identical to or very close to the problem from the text or lab.

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 3 points each. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [5.3/~Ex. 2] Give an expression in terms of a limit that gives the area under the curve $y = x^2$ on the interval $\left[2,5\right]$.

   (a) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(\frac{3k}{n}\right)^2\frac{5}{n}$ (b) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(2+\frac{k}{n}\right)^2\frac{3}{n}$ (c) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(2+\frac{k}{n}\right)^2\frac{5}{n}$

   (d) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2\frac{3}{n}$ (e) $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left(2+\frac{3k}{n}\right)^2\frac{3}{n}$

2. [5.3/Ex. 4] If $\int_{-2}^{1} f(x)\,dx = 3$ and $\int_{-2}^{7} f(x)\,dx = -5,$ what is the value of $\int_{1}^{7} f(x)\,dx$?

   (a) -2 (b) -8 (c) 8 (d) 2 (e) not enough information is given

3. [5.4/42] If $F(x) = \int_{x}^{2} \frac{e^t}{t}\,dt$, find the value of $F'(1)$.

   (a) $-e$ (b) e (c) $\frac{e^2}{2}$ (d) $-\frac{e^2}{2}$ (e) not defined

4. [5.1/Ex. 6] The graph of a certain function F has slope $4x^3 - 5$ at each point $\left(x, y\right)$ and contains the point $\left(-1, 2\right)$. Find the function F.

   (a) $F(x) = x^4 - 5x + 6$ (b) $F(x) = x^4 - 5x + 8$ (c) $F(x) = x^4 - 5x - 4$

   (d) $F(x) = 12x^2 - 5x$ (e) $F(x) = \left(4x^3 - 5\right)\left(x+1\right) - 2$

5. [4.1] Find values of *a* and *b* so that the function $f(x) = x^2 + ax + b$ has a local minimum at the point $(6, -5)$. Give the value of *b*.

(a) $b = 31$ (b) $b = 5$ (c) $b = -5$ (d) $b = -23$ (e) $b = -47$

6. [4.4] Let $f(x) = \dfrac{ax^4 + 5.1x^3 + \sqrt{2}x^2 - 7.2x - \pi}{bx^4 + 2\pi x^3 - \sqrt{5}x^2 - 13x + 143}$, where *a* and *b* are nonzero constants. If $\lim_{x\to\infty} f(x) = 2$, how are *a* and *b* related?

(a) $a = \dfrac{b}{2}$ (b) $a = \sqrt{b}$ (c) $a = b^2$ (d) $a = \dfrac{\pi}{2}b$ (e) $a = 2b$

7. [4.5/36] Find the limit: $\lim_{x\to 0^+} (\sin x)^{1/\ln\sqrt{x}}$.

(a) e (b) ∞ (c) $\dfrac{14e}{5}$ (d) e^2 (e) does not exist

8. [4.7/2] The cost *C*, in dollars, of producing *x* units of a particular commodity is $C(x) = \frac{2}{5}x^2 + 3x + 10$ and the selling price is $p(x)$ dollars per unit when *x* units are produced, where $p(x) = \frac{1}{5}(45 - x)$. How many units should be produced in order to maximize the profit?

(a) 3 (b) 5 (c) 7 (d) 10 (e) 13

9. [3.6] Given that $y^2 - 4xy + 5x = 6$ and y is a function of x, find y'.

(a) $\dfrac{4y-5}{4x+2y}$ (b) $\dfrac{4y-5}{2y}$ (c) $\dfrac{4y-5}{2y-4x}$ (d) $\dfrac{5-4y}{4x}$ (e) $\dfrac{5-2y}{2}$

10. [3.7] Let *x* and *y* be functions of *t*. Find $\dfrac{dx}{dt}$ where $x^2 + xy + 2y^2 = 2$ and $\dfrac{dy}{dt} = 2$ when $x = 1$ and $y = \dfrac{1}{2}$.

(a) -2.5 (b) $-\dfrac{12}{5}$ (c) 3 (d) 2 (e) $\dfrac{1}{2}$

11. [3.7] When a spherical balloon is inflated, the radius of the balloon is increasing at the rate of 0.25 cm/min, how fast is the volume changing when the radius is 4 cm? The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

(a) 16π cm^3/min (b) 8π cm^3/min (c) 4π cm^3/min
(d) $\frac{16}{3}\pi$ cm^3/min (e) $\frac{4}{3}\pi$ cm^3/min

12. [2.2/~17] Evaluate the limit: $\lim_{x\to 4} \frac{\sqrt{x}-a}{x-a^2}$, where a is a positive constant.

(a) does not exist (b) $\frac{1}{3a}$ (c) 0 (d) $\frac{1}{2}$ (e) $\frac{1}{2+a}$

13. [3.1] The limit $\lim_{x\to 1}\left(\frac{(2x^2+1)-3}{x-1}\right)$ represents

(a) $f'(3)$ where $f(x) = 2x^2+1$ (b) $f'(0)$ where $f(x) = 2x^2-2$
(c) $f'(1)$ where $f(x) = \frac{2x+1}{x}$ (d) $f'(1)$ where $f(x) = 2x^2+1$
(e) $f'(0)$ where $f(x) = \frac{2x-2}{x-1}$

14. [3.1] Suppose $f(x) = \sqrt{5x+4}$. Given that $f'(x) = \frac{5}{2\sqrt{5x+4}}$, find the equation of the tangent line to the graph of f at the point (1,3).

(a) $y = 3 + \frac{1}{2\sqrt{5x+4}}(5x-5)$ (b) $y = \frac{1}{6}(5x-9)$ (c) $y = \frac{1}{6}(5x+13)$
(d) $y = \frac{1}{6}(5x-2)$ (e) there is no such equation

15. [3.1/~51] Let $f(x) = \begin{cases} -2x & \text{if } x < 1 \\ 2-4\sqrt{x} & \text{if } x \ge 1 \end{cases}$. Find $f'(1)$, if it exists.

(a) $-\frac{2}{\sqrt{x}}$ (b) 0 (c) $-\frac{\sqrt{5}}{5}$ (d) -2 (e) does not exist

16. [3.2/~37] Find the *x*-coordinate of the point on the graph of $f(x)=\sqrt{x}\left(7x-6\right)$ where the tangent line is horizontal.

(a) $\frac{2}{7}$ (b) 0.3 (c) $\frac{1}{3}$ (d) $\frac{\sqrt{5}}{7}$ (e) $-\frac{1}{3}$

17. [3.5] Suppose *f* is a function with the property that $f'(x)=\cos\left(x^2\right)$. Find $g'(x)$, where $g(x)=f\left(x^3\right)$.

(a) $g'(x)=\cos\left(x^6\right)$ (b) $g'(x)=\sin\left(x^6\right)$ (c) $g'(x)=\cos\left(x^5\right)$

(d) $g'(x)=3x^2\cos\left(x^6\right)$ (e) undefined

INSTRUCTIONS FOR PART II: For these questions, you must write down **<u>all</u>** steps in your solutions **<u>as if you did not have a calculator</u>**. Write legibly and carefully label any graphs or pictures. **<u>Draw a box around your solution.</u>** Partial credit will be given for those parts of your solution that are correct.

18. **(10 pts)** [4.4/47] Sketch a carefully labeled graph of a function *f* with ALL the following properties:
   - the graph has $y=1$ and $x=3$ as asymptotes
   - $f'(x)>0$ for $x<3$ and $3<x<5$; $f'(x)<0$ for $x>5$
   - $f''(x)>0$ for $x<3$ and for $x>7$; $f''(x)<0$ for $3<x<7$
   - $f(0)=4$, $f(5)=4$ and $f(7)=2$

19. **(10 pts)** [3.7] A ladder 13 ft long rests against a vertical wall and is sliding down the wall at the rate of 2 ft/s at the instant the bottom of the ladder is 5 ft from the base of the wall. At this instant, how fast is the bottom of the ladder moving away from the wall?

20. **(9 pts)** [2.3/~39,~40] Find a constant *k* such that $f(x)=\begin{cases}5x+k & x\le -2\\ 2x & x>-2\end{cases}$ is continuous for all real numbers *x*. In particular, using your value of *k*, JUSTIFY why *f* is continuous on $\mathbf{R}$.

21. **(10 pts)** [3.6] Suppose that *f* is a function that is differentiable for all real *x*, and satisfies $f(e)=\ln 17$, $f'(e)=1-\frac{\ln 17}{e}$. Find $\frac{dy}{dx}$ at *e*, where $y=x^{f(x)}$.

22. **(10 pts)** [5.5/39] Evaluate $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$, showing all steps as if you do not have a calculator.