MATH 1426 **MIDTERM 1A**

FALL 2003

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question. On the scantron form, print your name legibly and indicate the final exam version (either A or B).

Each of the questions in this part counts 3 points each, for a total possible score of 45 points. You may use a calculator. You may write on this exam or request scratch paper if needed.

1. Find the difference quotient for $f(x)=\frac{1}{x}$ at $x=2$.

A. $\frac{-1}{2+\Delta x}$ B. $\frac{\Delta x}{2+\Delta x}$ C. $\frac{-1}{2(2+\Delta x)}$ D. $\frac{2}{2+\Delta x}$ E. $\frac{-1}{2}$

2. Find an equation of the line tangent to $f(x)=x^2+2x$ at the point (1,3).

A. $x-4y+11=0$ B. $4x+y=7$ C. $x+4y=13$ D. $y=4x$ E. $4x-y=1$

3. If $f(x)=\begin{cases} x^2+1 \text{ if } x<1 \\ 2x \text{ if } x\geq 1 \end{cases}$, then find $f'(1)$.

A. does not exist B. 2 C. 1 D. 0 E. ∞

4. If $f(x)=\frac{x}{\sqrt{x}+1}$, find $f'(4)$.

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{1}{9}$ D. $\frac{2}{9}$ E. $\frac{4}{3}$

5. Which of the following functions is continuous at the point (1,0) but not differentiable at (1,0)?

A. $f(x)=|x-1|$ B. $g(x)=x^{1/3}$ C. $h(x)=\frac{1}{x-1}$ D. $k(x)=|x|$

E. no such function exists

6. Find $f'(0)$ if $f(x)=\begin{cases} x^2 \text{ if } x\geq 0 \\ -x^2 \text{ if } x<0. \end{cases}$

A. 1 B. −1 C. 0 D. not enough information given

E. does not exist

7. Determine c so that the following function is continuous. Choose an interval that contains c.

$$f(x)=\begin{cases}2x-1 \text{ if } x<2\\ c+2 \text{ if } x=2\\ cx+1 \text{ if } x>2\end{cases}$$

A. $[-1,0]$ B. $[2,4]$ C. $\left[\frac{1}{2},\frac{3}{2}\right]$ D. $[4,5]$ E. $\left[-\frac{1}{2},\frac{1}{2}\right]$

8. Suppose $p(x)=\begin{cases}2-e^{-x} \text{ if } x\le 0\\ e^{x} \text{ if } x>0.\end{cases}$ Compute $\lim\limits_{h\to 0^+}\left(\dfrac{p(h)-p(0)}{h}\right)$.

A. 1 B. e^x C. 0 D. e^h E. e^{-h}

9. If f and g are differentiable functions such that $f(2)=3,\ f'(2)=-1$, and $f'(8)=5$ find $(4f)'(2)$.

A. 0 B. 5 C. 4 D. –4 E. not enough information given

10. If f and g are differentiable functions such that $f(2)=3,\ f'(2)=-1,\ g(2)=-5,$ and $g'(2)=2,$ find $\left(\dfrac{f}{g}\right)'(2)$.

A. $\frac{11}{25}$ B. $-\frac{1}{25}$ C. $-\frac{1}{2}$ D. 0 E. not enough information given

11. Find $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x}{x-1}$.

A. $+\infty$ B. $-\infty$ C. $-\pi$ D. does not exist E. 0

12. If $\log_{\sqrt{b}}106=2,$ what is $\sqrt{b-25}$?

A. 9 B. 2 C. 106 D. 25 E. 0

13. Solve for x: $\log_2\left(x^{\log_2 x}\right)=4$.

A. 4 B. $\frac{1}{4}$ C. no solutions D. $4,\frac{1}{4}$ E. ± 2

14. If f and g are continuous functions with $f(3)=5$ and $\lim_{x\to 3}\left[2f(x)-g(x)\right]=4,$ find $g(3)$.

A.0 B. 6 C. 3 E. 5 E. not enough information given

15. Find an equation of the normal line to the graph of $f(x)=\frac{1}{x}$ at $x=2$.

A. $x+4y-4=0$ B. $2x-8y-15=0$ C. $4x+y-4=0$ D. $x=-\frac{1}{4}$
E. $8x-2y-15=0$

INSTRUCTIONS FOR PART II: For these questions, you should write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 11 points, for a total possible score of 55 points.

16. Find the point(s) on the graph of $f(x)=x^2$ where the tangent line to the graph of $y=f(x)$ is parallel to the line $2x+y+3=0.$

17. Use the definition of the derivative to show that $f'(x)=\frac{1}{2\sqrt{x}}$ when $f(x)=\sqrt{x}.$

18. Use the quotient rule to determine the derivative of $f(x)=1+\frac{1}{\sqrt{x}}.$

19. Find an equation for the tangent line to the graph of $y=(x-1)\left(\sqrt[3]{x^{10}}+1\right)$ at the point with x-coordinate 1.

20. Evaluate $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$ without using a calculator or L'Hôpital's rule.