Math 1426

Spring 2004 **Midterm 1A**

The square brackets following an exam question number refer to a section/problem number in the text or a lab worksheet. Problems numbers preceded by the symbol ~ are modeled on that problem from the text or lab, but not identical to it. Problems numbers without the symbol are identical to or very close to the problem from the text or lab.

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 4 points each. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [3.1/14] Find the difference quotient for $f(x) = x^2 + x$.

   A. $x+1+\Delta x$ B. $\dfrac{2x}{\Delta x}$ C. $2x+1+\Delta x$ D. $\dfrac{2x\Delta x+(\Delta x)^2 - x}{\Delta x}$

   E. $\dfrac{2x\Delta x+\left(\Delta x\right)^2 + 2x + \Delta x}{\Delta x}$

2. [3.1/23] Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ where $f(x) = x^2 + x$.

   A. $2x+1$ B. 2 C. $3x+h$ D. 0 E. does not exist

3. [3.1] The derivative of $f(x)$ at a point $\left(c, f(c)\right)$ is given by

   A. $\dfrac{f(c+\Delta x)-f(c)}{\Delta x}$ B. $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ C. $\lim\limits_{x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

   D. $\lim\limits_{\Delta x\to 0}\dfrac{f(c+\Delta x)-f(c)}{\Delta x}$ E. $\lim\limits_{\Delta c\to 0}\dfrac{f(c+\Delta x)-f(c)}{\Delta x}$

4. [2.2/57] Find $\lim\limits_{x\to 3} f(x)$ if $f(x) = \begin{cases} 2(x+1), & x<3 \\ 4, & x=3 \\ x^2-1, & x>3 \end{cases}$

   A. 4 B. 8 C. 7 D. does not exist E. 3

5. [2.2/4] Evaluate the limit: $\lim_{x\to 0}\dfrac{\sin^{-1} x}{x-1}$

A. 1 B. -1 C. does not exist D. $\dfrac{\pi}{2}$ E. 0

6. [2.3/~16,20] The function $f(x)=\dfrac{x-3}{x-3}$ is discontinuous

A. at 0 only B. at $\frac{1}{3}$ only C. nowhere D. at 0 and 3 only E. at 3 only

7. [2.3/12] If $g(x)=\begin{cases} 2x+1 & \text{if } x\le -1 \\ 3x & \text{if } -1<x<1 \\ 2x-1 & \text{if } x\ge 1 \end{cases}$, then g is discontinuous at

A. 1 only B. 0 only C. -1 & 1 only
D. no point of the domain E. every point of the domain

8. [2.3,~34] Constants a and b are chosen such that the function

$$f(x)=\begin{cases} \dfrac{ax-6}{x-2} & \text{if } x\ne 2 \\ b & \text{if } x=2 \end{cases}$$

is continuous for all $x\in\mathbf{R}$. The product ab is
A. 9 B. 2 C. 6 D. 3 E. undefined

9. [2.4/~W4,3(a)] How many distinct real solutions are there to the equation $\log_x(2x-3)=2$?
A. 0 B. 1 C. 2 D. 3 E. 4

10. [3.2/Midterm Review] If f and g are differentiable functions such that $f(3)=2$, $f'(3)=3$, $g(3)=4$, and $g'(3)=-2$, find $(f\cdot g)'(3)$.
A. 16 B. -8 C. -16 D. 8 E. 12

11. [3.2/Midterm Review] If f and g are differentiable functions such that $f(3)=2$, $f'(3)=3$, $g(3)=4$, and $g'(3)=-2$, find $\left(\dfrac{f}{g}\right)'(3)$.

A. $\frac{1}{2}$ B. -1 C. $-\frac{1}{2}$ D. 2 E. 1

12. [3.1/~51] If $f(x)=\begin{cases} x^2+2, & \text{if} \quad x \le 1 \\ x+2, & \text{if} \quad x>1 \end{cases}$ then, at $x=1$, f is

A. differentiable but not continuous
B. continuous but not differentiable
C. not continuous and not differentiable
D. continuous and differentiable
E. not defined

INSTRUCTIONS FOR PART II: For these questions, you must write down **<u>all</u>** steps in your solutions as if you did not have a calculator. Write legibly and carefully label any graphs or pictures. **<u>Draw a box around your solution.</u>** Partial credit will be given for those parts of your solution that are correct.

13. **<u>9 points</u>** [3.2/49-52] Show that the function $f(x)=\frac{1}{2}x^2+3$ satisfies the equation $y'''+y''+y'=x+1.$

14. **<u>10 points</u>** [Chapter 2 Supplementary Review /49] Find a value for *a* so that $\lim_{x\to 0}\frac{\sqrt{ax+1}-1}{x}=1$. You should solve algebraically; no credit will be given for use of L'Hôpital's Rule.

15. **<u>9 points total</u>** Sketch clear, well-labeled graphs that depict functions *f*,*g*,*h* and *s* that satisfy the indicated conditions.

(a) [2.3/W3,1-5] *f* has domain [0,5] and is continuous on [0,5), but is not continuous on [0,5].

(b) [2.3/W3,1-5] *g* is not continuous at $x=3$, but if $g(3)=7$ is changed to $g(3)=4$, then *g* becomes continuous at $x=3$.

(c) [2.3] *h* is continuous on [0,7] except at $x=3$, *s* is continuous on [0,7] except at $x=3$, and $h(x)+s(x)=0$ for all *x* in [0,7] (sketch *h* and *s* on separate coordinate systems).

16. **<u>10 points</u>** [2.4/ W4,5] Suppose the world's population grows according to the formula $P(t)=P_0e^{rt}$, where *t* is the number of years since the population is P_0 and *r* is the growth rate. If the population is now 6.1 billion people and if it continues to grow according to the given formula with a growth rate of 1.14%, how long (to the nearest year) will it take before there is only 1 square yard of land per person? (The Earth contains approximately 1.68×10^{14} square yards of land and 1 billion = 10^9.)

17. **<u>10 points</u>** [3.2/43] Find an equation for the tangent line to the graph of the function $f(x)=x^4-2x+1$ that is parallel to the line $2x-y-3=1$.