**MATH 1426** **FALL 2003**

**MIDTERM 2 – VERSION A**

**INSTRUCTIONS FOR PART I:** Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 3 points each, for a total possible score of 45 points. You may use a calculator. You may write on this exam or request scratch paper if needed.

1. Find $\frac{dy}{dt}$ where $y = 6\sqrt{x} + x^2$ and $\frac{dx}{dt} = \frac{2}{19}$ when $x = 4$.

A. $\frac{13}{19}$ B. 1 C. $\frac{22}{19}$ D. $\frac{19}{2}$ E. 28

2. Find the differential: $d\left(\ln\left(\cos x\right)\right)$.

A. $\pm \cot x$ B. $\pm\left(\cot x\right)dx$ C. $-\tan x$ D. $-\left(\tan x\right)dx$ E. $\frac{1}{x}$

3. Let *f* be a function continuous on [0,10], $f'(x) > 0$ for $x < 3$ and $x > 5$, and $f'(x) < 0$ for $3 < x < 5$. Then

A. *f* has neither a relative maximum nor a relative minimum point.
B. *f* has a relative maximum but not a relative minimum point.
C. *f* has a relative maximum and a relative minimum point.
D. *f* has a relative minimum but not a relative maximum point.
E. It is not possible to determine whether *f* has a relative maximum or a relative minimum point.

4. Given $f(x) = \begin{cases} \frac{3-\sqrt{x}}{9-x}, & x \neq 9 \\ k, & x = 9 \end{cases}$, find the value of *k* such that *f* is continuous on **R**.

A. $-\frac{1}{6}$ B. $\frac{1}{6}$ C. $-\frac{1}{3}$ D. $\frac{1}{3}$ E. does not exist

5. The functions $f$ and $g$ and their first and second derivatives $f'$, $g'$, $f''$, and $g''$ are defined on **R**. At 0, 1 and 2 they take on the values given in the following table

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ | $f''(x)$ | $g''(x)$ |
|---|---|---|---|---|---|---|
| 0 | 2 | –3 | –4 | 7 | –2 | 0 |
| 1 | 0 | 1 | 10 | –2 | –4 | –5 |
| 2 | 6 | 5 | 5 | 3 | 3 | 4 |

If $H(x) = f(x)e^{3x}$, compute $H'(0)$.

A. –24 B. –2 C. 0 D. 2 E. 10

6. Referring to the table in Question 5, if $K(x) = \tan^{-1}\big(f(x)\big) = \arctan\big(f(x)\big)$, compute $K'(1)$.

A. –10 B. $\frac{1}{10}$ C. 1 D. 10 E. does not exist

7. Referring to the table in Question 5, find an equation of the tangent line to the graph of $y = f(x)$ at the point with $x$-coordinate 2.

A. $y = 5$ B. $x - 5y + 4 = 0$ C. $5x + y + 4 = 0$ D. $5x - y + 4 = 0$
E. $5x - y - 4 = 0$

8. Find all real numbers that satisfy $\ln(x-1) + \ln(x+1) = 2\ln\sqrt{12}$.

A. $\sqrt{13}$ B. $\sqrt{12}$ C. $\pm\sqrt{12}$ D. $\pm\sqrt{13}$ E. none of these

9. Let $f(x) = \begin{cases} -2x \text{ if } x < 1 \\ 2 - 4\sqrt{x} \text{ if } x \geq 1 \end{cases}$. Find $f'(1)$, if it exists.

A. –2 B. $-\frac{2}{\sqrt{x}}$ C. 0 D. $-\frac{\sqrt{5}}{5}$ E. does not exist

10. Find all critical number(s) of $6t^3 - 15t^2 + 8t + 4$.

A. $\frac{1}{\sqrt{10}}, \frac{4}{\sqrt{10}}$ B. $\frac{1}{\sqrt{10}}$ C. $\frac{3}{2}$ D. 0, 1 E. $\frac{1}{3}, \frac{4}{3}$

11. Find all critical number(s) of $x^{2/3}(5-2x)$.

A. 0, 3 B. 0,1 C. 1 D. 0 E. 3

12. Find the smallest and largest values of $x^4 - 2x^5 + 5$ on [0,1].

A. 4,5 B. 4, 5.00512 C. 3.9981, 5.01 D. $\frac{2}{5}$,1 E. $\frac{2}{5}$,5

13. Find the slope of the tangent line to the graph of $y = \frac{\sin x}{\sin x + \cos x}$ at $x = \frac{\pi}{4}$.

A. $\frac{1}{\sqrt{5}}$ B. 0.4998 C. $\frac{1}{2}$ D. $\sqrt{2}$ E. 0.5002

14. Which of the following functions satisfy $y'' + 4y = 0$?

A. $\sin x$ B. $\cos x$ C. $\sin 2x$ D. $\sin 4x$ E. $\cos 4x$

15. A basketball is dropped from the height of 6 feet. Use the formula $s(t) = -16t^2 + 6$ for the position function, *t* in seconds. How long (in seconds) does it take for the ball to be at the height of 2 feet?

A. 2 B. 4 C. $\frac{1}{16}$ D $\frac{1}{4}$ E. $\frac{1}{2}$

**INSTRUCTIONS FOR PART II:** For these questions, you **must** write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. Partial credit will be given for those parts of your solution that are correct. **Draw a box around your final answer.** Each of the questions in this part counts 11 points, for a total possible score of 55 points.

16. A ladder 13 ft long rests against a vertical wall and is sliding down the wall at the rate of 3 ft/sec at the instant the foot of the ladder is 5 ft from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

17. Let *f* be a function continuous on [0,8], $f'(x) > 0$ and $f''(x) > 0$ for $x < 2$ and $x > 4$, and $f'(x) < 0$ and $f''(x) > 0$ for $2 < x < 4$. Sketch the graph of $y = f(x)$, labeling any relative extrema and any inflection points.

18. Find an equation for the tangent line to the graph of the function $y = \sin(\sin x)$ at the point with $x$-coordinate $\pi$.

19. For the function $y = y(x)$, if $xy^3 + xy = 6$, and $y(3) = 1$, find $y'(3)$.

20. A box with an open top and a square base is to be built so that the height of the box plus the length of one of the sides is to be 8 meters. Find the **EXACT** dimensions for such a box that yield the maximal volume; justify your reasoning, showing all steps in your solution.