Math 1426

Spring 2004 **Midterm 2A**

The square brackets following an exam question number refer to a section/problem number in the text or a lab worksheet. Problems numbers preceded by the symbol ~ are modeled on that problem from the text or lab, but not identical to it. Problems numbers without the symbol are identical to or very close to the problem from the text or lab.

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 4 points each. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. [3.7] Let x and y be functions of t. Find $\frac{dx}{dt}$ if $x^2 + xy + 2y^2 = 2$ and $\frac{dy}{dt} = 2$ when $x = 1$ and $y = \frac{1}{2}$.

   (a) $\frac{9}{16}$ (b) 3 (c) 2 (d) $\frac{1}{2}$ (e) $-\frac{12}{5}$

2. [3.7] The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. When a spherical balloon is inflated, the radius of the balloon is increasing at the rate of 0.25 cm/min, how fast is the volume changing when the radius is 4 cm?

   (a) 16π cm^3/min (b) 8π cm^3/min (c) 4π cm^3/min
   (d) $\frac{16}{3}\pi$ cm^3/min (e) $\frac{4}{3}\pi$ cm^3/min

3. [3.8] Find the differential $d(x^2 \sin 2x)$

   (a) $2x\sin 2x dx - 2x^2 \cos 2x dx$ (b) $(2x\sin 2x + x^2 \cos 2x)dx$
   (c) $2x(\sin 2x + x\cos 2x)$ (d) $2x\sin 2x + x^2 \cos 2x$
   (e) $2x(\sin 2x + x\cos 2x)dx$

4. [4.1/Ex. 7] Find all critical number(s) of $f(x) = x^{2/3}(5 - 2x)$ on the interval $[-1, 2]$.

   (a) 0 (b) 1 (c) 0,1 (d) $\frac{5}{2}$ (e) $0, \frac{5}{2}$

5. [3.3/15] Let $f(x) = x^2 \ln x$. The first step in finding the derivative of $f(x)$ is

   (a) $f(x) = \ln x^3$ and $\dfrac{d}{dx}\left(\ln x^3\right) = \dfrac{1}{\dfrac{d}{dx}\left(x^3\right)} \cdot x^3$

   (b) $f'(x) = x^2 \dfrac{d}{dx}\left(\ln x\right) + \ln x \dfrac{d}{dx}\left(x^2\right)$ with $\dfrac{d}{dx}\left(\ln x\right) = \dfrac{1}{x}$ and $\dfrac{d}{dx}\left(x^2\right) = 2x$

   (c) $f'(x) = \dfrac{d}{dx}\left(x^2\right)\dfrac{d}{dx}\left(\ln x\right)$ with $\dfrac{d}{dx}\left(\ln x\right) = \dfrac{1}{x}$ and $\dfrac{d}{dx}\left(x^2\right) = 2x$

   (d) $f'(x) = x^2 \dfrac{d}{dx}\left(\ln x\right) - \ln x \dfrac{d}{dx}\left(x^2\right)$ with $\dfrac{d}{dx}\left(\ln x\right) = \dfrac{1}{x}$ and $\dfrac{d}{dx}\left(x^2\right) = 2x$

   (e) $f'(x) = x^2 \dfrac{d}{dx}\ln x + \ln x \dfrac{d}{dx}\left(x^2\right)$ with $\dfrac{d}{dx}\left(\ln x\right) = \dfrac{1}{2}\ln x^2$ and $\dfrac{d}{dx}\left(x^2\right) = 2x$

6. [3.4/19] Assume that the position at time *t* of an object moving along a line is given by $s(t) = t^3 - 9t^2 + 15t + 25$ on the interval [0,6]. The total distance traveled by the object during the indicated time interval is

   (a) 7 (b) 25 (c) 32 (d) 46 (e) 49

7. [2.2] Evaluate the limit: $\lim\limits_{x \to 0} \dfrac{\sin^2 ax}{bx}$, where *a* and *b* are nonzero constants.

   (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) does not exist (e) ∞

8. [4.2]If you drive from here to Dallas, which theorem implies that your average speed for the whole journey will equal your instantaneous speed attained at some moment(s) during your journey?

   (a) the mean-value theorem (b) the intermediate-value theorem
   (c) the squeeze theorem (d) the extremal-value theorem
   (e) the constant-difference theorem

9. [4.3]You are given $f(x) = -3x^3 + ax^2 + b$ where *a* and *b* are nonzero constants. Find the value of *a* that gives *f* a relative extremum at $x = 2$.

   (a) 2 (b) 9 (c) $\frac{2}{9}$ (d) 18 (e) no such value

10. [4.3] For every function *f*, the *definition* of an inflection point on the graph of *f* is a point $\left(a, f(a)\right)$ where

    (a) $f'(a)$ is defined but $f''(a)$ is not defined (b) $f'(a)$ is defined and $f''(a) = 0$
    (c) $f'(a) = 0 = f''(a)$ (d) the concavity of the graph changes
    (e) both $f'(a)$ and $f''(a)$ are not defined

For problems 11 and 12, refer to the table below.

x	$f(x)$	$f'(x)$
-1	0	-2
0	-2	1
1	1	2

11. If $g(x) = f(x)\cos 2x$, compute $g'(0)$.
    (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

12. If $h(x) = e^{f(x)}$, compute $h'(-1)$.
    (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

13. [4.3/~12,~14,~16,~17] A function f has domain **R** and derivative $f'(x) = x^3(5x-8)$. Find the critical numbers of f and identify which type of extremum (if any) each one yields.
    (a) there is a relative minimum at 0 and a relative maximum at $\frac{8}{5}$
    (b) there is a relative maximum at 0 and a relative minimum at $\frac{8}{5}$
    (c) there is a relative maximum at 0; $\frac{8}{5}$ is a critical number which is not a relative extremum
    (d) there is a relative minimum at 0; $\frac{8}{5}$ is a critical number which is not a relative extremum
    (e) there is a relative maximum at $\frac{8}{5}$; 0 is a critical number which is not a relative extremum

INSTRUCTIONS FOR PART II: For these questions, you must write down **all** steps in your solutions as if you did not have a calculator. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct.

14. **(10 pts)** [3.7] Two people start from the same point. One walks east at 3 mile/hour and the other walks north at 2 mile/hour. How fast is the distance between the people changing after 15 minutes?

15. **(10 pts)** Find an equation for the tangent line to the graph of the equation $\sin(x-y) = xy$ at the point $(0, \pi)$.

16. **(9 pts)** [4.1/Ex. 9] A box with a square base is constructed so that the length of one side of the base plus the height is 10 in. What is the largest possible volume of such a box?

17. **(9 pts)** Sketch the graph of a function f with ALL the following properties:
$f(-1) = 4, \quad f(1) = 0, \quad f'(-1) = 0, \quad f'(1)$ does not exist,
$f'(x) < 0$ if $|x| < 1, \quad f'(x) > 0$ if $|x| > 1, \quad f''(x) < 0$ if $x \neq 1$.
What can be said about the inflection points of f? Explain.

18. **(10 pts)** [3.4/41] A person standing at the edge of a cliff throws a rock directly upward. It is observed that 2 seconds later the rock is at its maximum height (in ft) and that 5 seconds after that, it hits the ground at the base of the cliff. The height of the rock is given by the formula
$$h(t) = -16t^2 + v_0 t + s_0,$$
where v_0 is the initial velocity and s_0 is the initial position.
(a) Find the height of the cliff.
(b) Determine the velocity of the rock when it hits the ground.