

MIDTERM II PRACTICE QUESTIONS

PART I.

1.[3.1, 3.2] Find $f'(0)$ for $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ (x-1)^2 & \text{if } x \geq 0 \end{cases}$

- A. 0 B. 1 C. -1 D. -2 E. does not exist

2.[2.2] Find the limit: $\lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin 2x}$

- A. 1 B. $-\infty$ C. -1 D. 0 E. does not exist

3.[2.2] Find the limit: $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

- A. $\frac{1}{\sqrt{3}}$ B. $\frac{\sqrt{3}}{6}$ C. $\frac{\sqrt{3}}{3}$ D. 0 E. does not exist

4.[2.2, 3.1] Suppose that a function f is differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{f(1+h)}{h} = 6$.

Find $f(1) + f'(1)$

- A. 6 B. 1 C. 2 D. 0 E. 5

5. [3.5] Suppose f is a function with the property that $f'(x) = \cos(x^2)$. Find $g'(x)$, where $g(x) = f(x^2)$.

- A. $g'(x) = 2x \cos(x^4)$ B. $g'(x) = \cos(x^4)$ C. $g'(x) = \sin(x^4)$
D. undefined. E. none of these.

6. [3.1] Interpret $\lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right)$ as a derivative.

- A. $\frac{d}{dx}(\cos 0)$ B. $\cos'(x)$ C. $\cos'(h)$ D. $\cos'(0)$ E. $\cos'(1)$

7. [2.4] Solve $\log_7(2x+3) = \log_7 3 + \log_7 11$

- A. $\log_7 15$ B. 7^{15} C. $\ln 15$ D. 15 E. None of these.

8. [3.6] The equation of the tangent line to the curve of function $y^2 = x^3(2-x)$ at $(1, 1)$ is

- A. $x + y = 0$ B. $y = -x + 1$ C. $x - y + 2 = 0$ D. $y = x$
E. $x + y + 1 = 0$

9. [3.2, 3.3] Find the derivative of $f(x) = \frac{1-2x}{e^x}$

- A. $\frac{xe^x - 2e^x}{e^x}$ B. $\frac{2xe^x - 3e^x}{e^{2x}}$ C. $\frac{xe^x - 2e^x}{e^{2x}}$ D. $\frac{2xe^x - 3e^x}{e^x}$
E. None of these.

10.[3.5] Find the derivative $\frac{d}{dx}\left[(4x^3 + 2\sqrt{x})^2\right]$

A. $2(4x^3 + 2\sqrt{x})(12x^2 + \frac{2}{\sqrt{x}})$ B. $(4x^3 + 2\sqrt{x})^2(12x^2 + \frac{1}{\sqrt{x}})$

C. $(4x^3 + 2\sqrt{x})(12x^2 + \frac{1}{\sqrt{x}})$ D. $2(4x^3 + 2\sqrt{x})(12x^2 + \frac{1}{\sqrt{x}})$

E. None of these.

11. [3.2, 3.5] Find the derivative of $f(x) = (x-1)^3(\sqrt[3]{x^{10}} + 1)$ at $x = 0$.

A. 3 B. 1 C. $-\frac{1}{3}$ D. -1 E. none of these.

12. [3.5, 3.6] Find the derivative of $y = (3x-1)^2 \sin^{-1}(x^2)$

A. $6(3x-1)\sin^{-1}(x^2) + 2x(3x-1)^2/(1+x^2)$ B. $(3x-1)^2 \cos^{-1}(x^2) + 6(3x-1)\sin^{-1}(x^2)$

C. $6(3x-1)\sin^{-1}(x^2) + 2x(3x-1)^2/(1+x^4)$ D. $2(3x-1)[3\sin^{-1}(x^2) + x(3x-1)/\sqrt{1-x^2}]$

E. $2(3x-1)[3\sin^{-1}(x^2) + x(3x-1)/\sqrt{1-x^4}]$

13. [3.1]The slope of the tangent line to the graph of $y = x + \frac{4}{x}$ at $x = 2$ is

A. $1 - 4x^{-2}$ B. 0 C. 1 D. does not exist E. none of these.

14. [3.6] Given that $x^y = y^x$, compute $\frac{dy}{dx}$ at the point (2,2).

A. $\ln 2$ B. 1 C. 2 D. $\frac{1}{2}$ E. none of these.

15. [3.2, 3.5] Find the derivative of $y = \frac{(1+2x)^{3/2}}{(1+3x)^{4/3}}$

A. $\frac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\frac{3}{2(1+2x)} - \frac{4}{3(1+3x)}\right)$ B. $\frac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\frac{3}{2(1+2x)} + \frac{4}{3(1+3x)}\right)$

C. $\frac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\frac{3}{1+2x} - \frac{4}{1+3x}\right)$ D. $\frac{(1+2x)^{3/2}}{(1+3x)^{4/3}}\left(\frac{3}{1+2x} + \frac{4}{1+3x}\right)$

E. None of these.

16. [3.6] Find $\frac{dx}{dt}$ where $x^2 + xy + 2y^2 = 2$ and $\frac{dy}{dt} = 2$ when $x = 1$ and $y > 0$

A. 3 B. $\frac{19}{6}$ C. $-\frac{12}{5}$ D. 2 E. $\frac{1}{2}$

17. [3.7] Find $\frac{dy}{dt}$ where $y = 6\sqrt{x} + x^2$ and $\frac{dx}{dt} = \frac{2}{19}$ when $x = 4$

A. 1 B. $\frac{19}{2}$ C. $\frac{13}{19}$ D. 28 E. $\frac{22}{19}$

18. [3.8] Find the differential: $d(\cos x \tan 2x)$

- A. $(\sin x \tan 2x + 2 \cos x \sec^2 2x)dx$ B. $(-\sin x \tan 2x + 2 \cos x \sec^2 2x)dx$
C. $(-\sin x \tan 2x + 2 \cos x \csc^2 2x)dx$ D. $(\sin x \tan 2x + 2 \cos x \csc^2 2x)dx$
E. $(-\sin x \tan 2x + 2 \cos x \sec 2x)dx$

19 [4.1] The total number of critical numbers of function $f(x) = \frac{\ln \sqrt{x}}{x}$ on $[1, 3]$ is

- A. 1 B. 0 C. 2 D. 3 E. 4

20 [4.3] Find values of a and b so that the function $f(x) = x^2 + 2ax + b$ has a relative minimum at point $(3, 0)$.

- A. $a = -3, b = 0$ B. $a = -6, b = -3$ C. $a = -1, b = 2$
D. $a = -3, b = 9$ E. $a = 6, b = 9$

21. [3.8] Suppose that we know that a function g has derivative $g'(x) = \sqrt{x^2 + 16}$ for all x , and that $g(3) = -2$. Use a differential approximation (tangent line approximation) to estimate the value of $g(3.05)$.

- A. -2.01 B. -1.75 C. -1.95 D. -1.9 E. none of these.

22. [3.8] The edge of a cube was found to be 20 cm, with a possible error in measurement of 0.2 cm. What is the maximal possible error in computing the volume of the cube?

- A. 240 cm^3 B. 270 cm^3 C. 0.008 cm^3 D. too negligible to compute. E. none of these.

23. [4.1, 4.6] Let x and y be two positive numbers whose product is 100, to make $x + y$ a minimum,

- A. $x=20, y=5$ B. $x=5, y=20$ C. $x=-10, y=-10$ D. $x=-20, y=-5$
E. $x=10, y=10$

24. [4.1] Which one of the following statements is true?

- A. An absolute maximum is always a local maximum.
B. An absolute minimum is always a local minimum.
C. A local maximum can be an absolute maximum.
D. The end-point of an interval can be a local maximum.
E. None of the above.

25. [4.1] At a critical number c of a function f :

- A. $f'(c) = 0$ or f is not differentiable.
B. $f'(c) = 0$ or f is discontinuous.
C. f is discontinuous and not differentiable.
D. $f''(c) = 0$
E. None of the above.

26. [4.3] If $f'(x) = x^2(x-2)(5x-6)$, then f has a relative maximum at

- A. 0 B. $\frac{6}{5}$ C. 2 D. does not exist E. not enough information.

27. [4.1] The total number of critical numbers of function $f(x) = |x-2|$ on $[-3, 3]$ is

- A. 0 B. 1 C. 2 D. 3 E. 4

28. [4.1] If $s(t) = 1 - 2t - t^2$, then the absolute minimum of s over the interval $-4 \leq t \leq 1$ is
 A. -7 B. -2 C. -1 D. 2 E. does not exist.

29. [4.1] All the critical numbers of $g(t) = 5t^{2/3} + t^{5/3}$ are
 A. -2 & 0 B. -2 & 1 C. 0 & 1 D. -2 E. 1

PART II.

1. [2.3] Without using a calculator, find the correct value of k that makes the function $f(x)$ continuous on $[0, 11]$, if f is defined as follows:

$$f(x) = \begin{cases} k \cdot \sin \frac{(x+3)\pi}{6}, & x \leq 2 \\ \frac{3 - \sqrt{11-x}}{x-2}, & x > 2 \end{cases}$$

2. [3.2, 3.5] Two functions, f and g , are continuous and differentiable for all real numbers. Some values of the functions and their derivatives are shown in this table:

x	0	1	2	3	4
$f(x)$	1/2	1/3	1	4	3
$g(x)$	-2	1	3	2	-1/3
$f'(x)$	3/2	5/3	1/4	0	-4/5
$g'(x)$	-1	2/3	4	-3	-1/3
$f''(x)$	3	2	-1/3	2	1
$g''(x)$	1	0	3	0	1

Based on that luscious table, find the following derivatives:

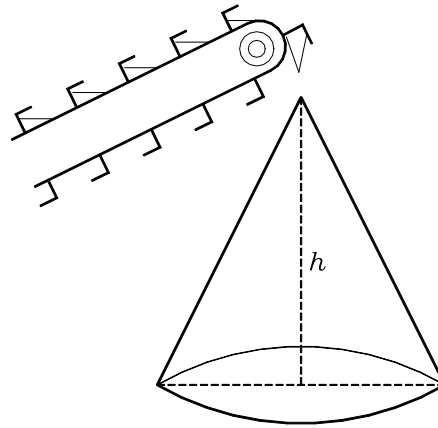
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|--|---|
| (a) $\frac{d}{dx} f(2x)$, evaluated at $x = 1$ | (e) Let $h(x) = f(g'(x))$, find $h'(2)$ |
| (b) $\frac{d}{dx} (f(x)g(x))$, evaluated at $x = 1$ | (f) Let $R(x) = f'(g(2x))$, find $R'(1)$ |
| (c) $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right)$, evaluated at $x = 0$ | (g) Let $r(x) = \ln[f(2x)]$, find $r'(2)$ |
| (d) Let $H(x) = f[g(x)]$, find $H'(1)$ | (h) Let $t(x) = f(x)^{g(x)}$, find $t'(3)$ |

3. [2.4] Find the solution(s) to the equation $(\ln y)^2 - 2 \ln(y^5) = 11$

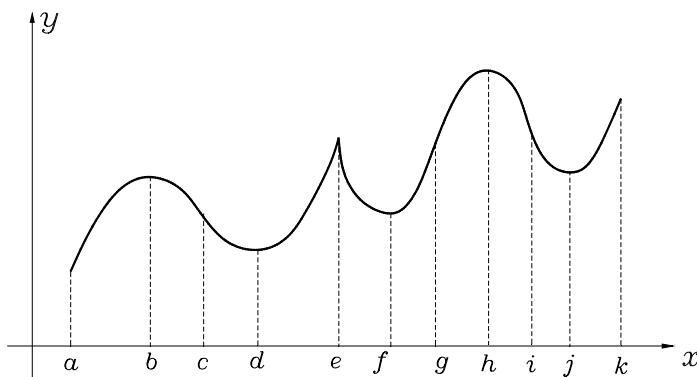
4. [3.7] Two people start from the same point. One walks east at 3 mile/hour and the other walks northeast at 2 mile/hour. How fast is the distance between the people changing after 15 minutes?

5. Find what values of constants a and b does $y = ae^x + bx \sin x$ satisfy $y'' + y = \cos x$?
6. [3.6] Find y' from the following implicit functions. (a) $x \sin(2y) - y \cos x = 2x$; (b) $x^2 y^2 = x + y$
7. [3.4] The position of a particle is given by the equation $s(t) = 10 + 64t - 16t^2$, where t is measured in seconds and s in ft. (a) Find the velocity $v(t)$ and acceleration $a(t)$ at time t . (b) What is the velocity and acceleration after 3 seconds? (c) How far does the particle travel after 3 seconds? (d) What is the total distance traveled after 3 seconds?

8. [3.7] Gravel is being dumped from a conveyor belt at a rate of $40 \text{ ft}^3/\text{min}$ and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. (a) How fast is the height of the pile increasing when the pile is 10 ft high? (b) At what rate is gravel being dumped if the height of the pile is increasing at a rate of $\frac{2}{\pi} \text{ ft/min}$ when it is 10 ft high? (The volume V of a cone is $V = \frac{1}{3} \pi r^2 h$)



9. [4.1,4.3] Identify the following points on the graph (a) All critical numbers; (b) All relative maximum; (c) All relative minimum; (d) The absolute maximum; (e) The absolute minimum; (f) Inflection points; (g) Intervals of increasing; (h) Intervals of decreasing;



10. [4.4] Find all vertical and horizontal asymptotes of the graph of each function. Find where each graph is rising and where it is falling, determine concavity, and locate all critical points and points of inflection.

(a) $f(x) = \frac{x^3 + 1}{x^3 - 8}$ (b) $f(x) = 1 - \frac{x}{4 - x}$ (c) $f(x) = \tan^{-1} x^2$

11. [3.7] At noon on a certain day, a truck is 250 mile due east of a car. The truck is traveling west at a constant speed of 25 mph, while the car is traveling north at 50 mph.
- a) At what rate is the distance between them changing at time t ?
- b) At what time is the distance between the car and the truck neither increasing nor decreasing?

You should also look over your lecture notes, homework assignment, problem-solving labs, quizzes, and Review questions for midterm 1. The Midterm 2 of Fall 2003 and Spring 2004 can be found at <http://www.uta.edu/math/pages/main/oldexams/calc1/calc1.htm>

Midterm 2: Friday, Nov. 3, 6:00 – 8:00 pm

Answers to PART I:

1.E 2.D 3.B 4.A 5.A 6.D 7..D 8.B 9.B 10.D 11.A 12.E 13.B 14.B 15.C 16.C 17.A 18.B 19.A 20.D 21.B 22.A 23.E 24.C 25.A 26.B 27.B 28.A 29.A

Answers to PART II:

1. $k = 1/3$; 2.(a) $-17/15$ (b) $17/9$ (c) $-5/8$ (d) $10/9$; (e) $-12/5$; (f) 16; (g) $-8/15$; (h) $-48\ln 4$;

3. e^{11} or $1/e$; 4. 3.9 mph; 5. $a = 0$, $b = 1/2$;

6.(a) $\frac{\sin 2y + y \sin x - 2}{\cos x - 2x \cos 2y}$ (b) $-\frac{1 - 2xy^2}{1 - 2x^2y}$

7.(a) $v(t) = 64 - 32t$, $a(t) = -32$ (b) $v(3) = -32$ ft/s, $a(3) = -32$ ft/s² (c) = 58 ft (d) 80 ft

8. (a) $\frac{8}{5\pi}$ (b) 50 ft³/min

9.(a) b, d, e, f, h, j; (b) b, e, h; (c) d, f, j; (d) h; (e) a; (f) c, g, i; (g) [a, b], [d, e], [f, h], [j, k]; (h) [b, d], [e, f], [h, j];

10. (a) vertical asymptote: $x = 2$; horizontal asymptote: $y = 1$; critical numbers: 0, 2; second critical numbers: $-\sqrt[3]{4}$, 0, 2;

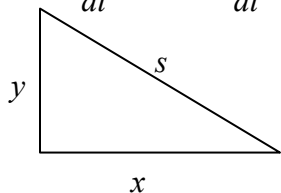
x	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, +\infty)$
$f'(x)$	< 0	$= 0$	< 0	$= 0$	< 0
	↘	—	↘	—	↘

intervals where f is rising: none; intervals where f is falling: $(-\infty, 2)$ and $(2, \infty)$;

X	$(-\infty, -\sqrt[3]{4})$	$-\sqrt[3]{4}$	$(-\sqrt[3]{4}, 0)$	0	$(0, 2)$	2	$(2, +\infty)$
$f''(x)$	< 0	$= 0$	> 0	$= 0$	< 0	$= 0$	> 0
Concavity	∩		∪		∩		∪

Inflection points: $-\sqrt[3]{4}$, 0, 2;

11. $\frac{dx}{dt} = -25$, $\frac{dy}{dt} = 50$, $s^2 = x^2 + y^2$, $\frac{ds}{dt} = ?$ (Related rates problem)



(a) $2s \frac{ds}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$, therefore $\frac{ds}{dt} = (x \frac{dx}{dt} + y \frac{dy}{dt}) / \sqrt{x^2 + y^2}$

(b) Set $\frac{ds}{dt} = 0$, we must have $x \frac{dx}{dt} + y \frac{dy}{dt} = 0$, that is

$-25x + 50y = 0$. Since $x = 250 - 25t$, $y = 50t$, one gets

$-25(250 - 25t) + 50(50t) = 0$, and $t = 2$ (hour)