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Fall 2003 M1426
Midterm2 (Solutions)

1. $y = 6\sqrt{x} + x^2$ and $\frac{dx}{dt} = \frac{2}{19}$ when $x = 4$

Find $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \left(6\left(\tfrac{1}{2}\right)\frac{1}{\sqrt{x}} + 2x\right)\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=4} = \left(3\,\frac{1}{\sqrt{4}} + 2(4)\right)\left(\frac{2}{19}\right)$$

$$= \left(\frac{3}{2} + 8\right)\left(\frac{2}{19}\right) = \frac{3}{19} + \frac{16}{19} = \frac{19}{19}$$

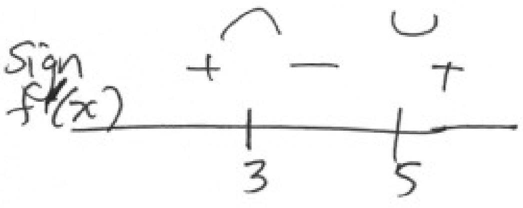
$$= \boxed{1} \quad \boxed{\text{ANS B}}$$

2. $$d(\ln(\cos(x))) = \frac{1}{\cos(x)}(-\sin x)\,dx$$

$$= \boxed{-\tan x \, dx}$$

$\boxed{\text{ANS D}}$

3. Make a quick signchart



$\boxed{C}$

Pg 2

④ $f(x) = \begin{cases} \dfrac{3-\sqrt{x}}{9-x}, & x \neq 9 \\ K, & x = 9 \end{cases}$ ⇐ Note here the only problem place is $\underline{\underline{x=9}}$

For continuity we need:

① $f(9)$ defined (it is: $f(9) = k$)

② $\lim\limits_{x \to 9^+} f(x) = \lim\limits_{x \to 9^-} f(x)$

$$\lim_{x \to 9^+} \frac{3-\sqrt{x}}{9-x} = \lim_{x \to 9^+} \frac{(3-\sqrt{x})(3+\sqrt{x})}{(9-x)(3+\sqrt{x})} \quad \text{[rationalized]}$$

$$= \lim_{x \to 9^+} \frac{(9-x)}{(9-x)(3+\sqrt{x})}$$

$$= \lim_{x \to 9^+} \frac{1}{3+\sqrt{x}} = \frac{1}{6}$$

(Note that $\lim\limits_{x \to 9^-} f(x) = \frac{1}{6}$ also)

③ $\lim\limits_{x \to 9} f(x) = f(9)$ (Need this for continuity)

So Solve: $\frac{1}{6} = f(9)$

$\boxed{\frac{1}{6} = K}$ $\boxed{\text{ANS B}}$

Note that you could see this on your graphing calc.

5. If $H(x) = f(x)e^{3x}$

Then $H'(x) = f'(x)e^{3x} + (f(x))(3e^{3x})$

So $H'(0) = f'(0)(1) + f(0)(3)(1)$

$= (-4)(1) + 2(3)$

$= \boxed{2}$ $\boxed{\text{ANS D}}$

6. $K(x) = \arctan(f(x))$

Find $K'(1)$ $\quad \dfrac{dK(x)}{dx} = \dfrac{1}{1+(f(x))^2} f'(x)$

$$\left.\frac{dK}{dx}\right|_{x=1} = \frac{1}{1+(f(1))^2}\left[f'(1)\right]$$

$$= \frac{10}{1+(0)} = \boxed{10} \quad \boxed{\text{ANS D}}$$

7. $y = f(x)$, $x = 2$ Find tangent line

from table

pt $(2, f(2)) = (2, 6)$

slope $f'(2) = 5$ (from table)

$y - 6 = 5(x-2)$

$y = 5x - 10 + 6$ $\rightarrow y = 5x - 4$

$\boxed{\text{ANS E}}$

⑧ $\ln(x-1) + \ln(x+1) = 2\ln\sqrt{12}$

$$\ln(x^2-1) = 2\ln\sqrt{2}$$

$$\ln(x^2-1) = 2\left(\tfrac{1}{2}\right)\ln 12$$

$$\ln(x^2-1) = \ln 12$$

$$e^{\ln(x^2-1)} = e^{\ln 12}$$

$$x^2-1 = 12$$

$$x^2 = 13 \qquad \boxed{\text{ANS. D}}$$

$$x = \pm\sqrt{13}$$

⑨ $f(x) = \begin{cases} -2x & \text{if } x<1 \\ 2-4\sqrt{x} & \text{if } x \geq 1 \end{cases}$ ~~Check~~

$$f'(x) = \begin{cases} -2 & \text{if } x<1 \\ -2/\sqrt{x} & \text{if } x>1 \end{cases}$$

Note $\lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^-} f'(x)$ [Shortcut]

Thus $f'(1) = -2$ $\boxed{\text{ANS A}}$

⑩ Find C.N. $f(t)=6t^3-15t^2+8t+4$

$$f'(t)=18t^2-30t+8$$

$f'(t)=0$ $\quad 18t^2-30t+8=0$

$$9t^2-15t+4=0$$

$\boxed{\text{ANS E}}$

Quadratic formula $\rightarrow$ $t=\dfrac{15\pm\sqrt{(15)^2-4(9)(4)}}{2(9)}$

$$=\frac{15\pm\sqrt{81}}{18}$$

$$=\frac{15\pm 9}{18}=\frac{24}{18} \text{ or } \frac{6}{18}$$

$$\doteq \boxed{\frac{4}{3} \text{ or } \frac{1}{3}}$$

⑪ Find C.N.

$$f(x)=x^{2/3}(5-2x)$$

$$f'(x)=\frac{2}{3}x^{-1/3}(5-2x)+x^{2/3}(-2)$$

$$=\frac{2(5-2x)+(-2)(3)(x)}{3x^{1/3}}$$

$$=\frac{10-4x-6x}{3x^{1/3}}=\frac{10(1-x)}{3x^{1/3}}$$

$x=0,\ x=1$ C.N. $\overset{\text{ANS}}{\boxed{B}}$

$\boxed{12.}$ $f(x) = x^4 - 2x^5 + 5$ on $[0,1]$

$f'(x) = 4x^3 - 10x^4 = x^3(4-10x)$

C.N. $x=0$, $4-10x=0$

$4 = 10x$

$.4 = x$

Check value at endpts

$f(0) = 5$

$f(1) = \boxed{4}$ smallest $\boxed{\text{ANS. B}}$

Check at C.N.

$f(0)=5$

$f(.4) = \boxed{5.00512}$ largest

3.) $y = \dfrac{\sin x}{\sin x + \cos x}$ at $x = \frac{\pi}{4}$

$$y' = \frac{(\sin x + \cos x)(\cos x) - \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$y'\Big|_{x=\frac{\pi}{4}} = \frac{(\sqrt{2})(\frac{\sqrt{2}}{2}) - 0}{(\sqrt{2})^2} = \boxed{\frac{1}{2}} \quad \boxed{\text{ANS C}}$$

⑭ Which satisfy $y'' + 4y = 0$

$y = \sin 2x$

ANS C

$y' = 2\cos 2x$

$y'' = -4\sin 2x$

So, $y'' + 4y = 0$

That is $-4\sin 2x + 4\sin 2x = 0$ ✓

⑮ $s(t) = -16t^2 + 6$

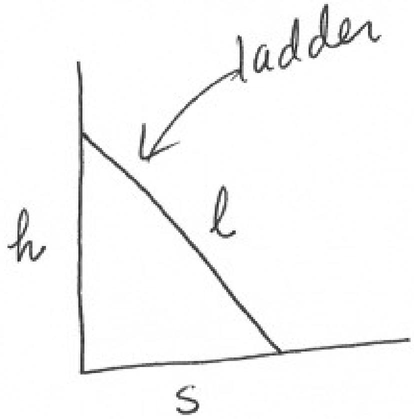
$2 = -16t^2 + 6$

$-4 = -16t^2$

$\frac{1}{4} = t^2 \quad t = \frac{1}{2}$

ANS E

⑯



$l = 13\text{ ft}$

Given $\left.\dfrac{dh}{dt}\right|_{s=5} = -3\text{ ft/sec}$

Want $\boxed{\dfrac{ds}{dt}}$ when $s=5$

$$l^2 = h^2 + s^2$$

$$13^2 = h^2 + s^2$$

$$169 = h^2 + s^2$$

use this to get $h(5)$

$$169 = h^2 + 5^2$$

$$144 = h^2$$

$$12 = h$$

$$s^2 = 169 - h^2$$

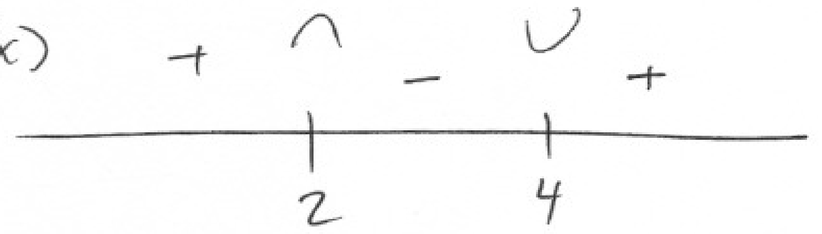
$$2s\frac{ds}{dt} = -2h\frac{dh}{dt}$$

$$\left.\frac{ds}{dt}\right|_{s=5} = \left.-\frac{h}{s}\frac{dh}{dt}\right|_{s=5} = -\frac{h(5)}{5}(-3)$$

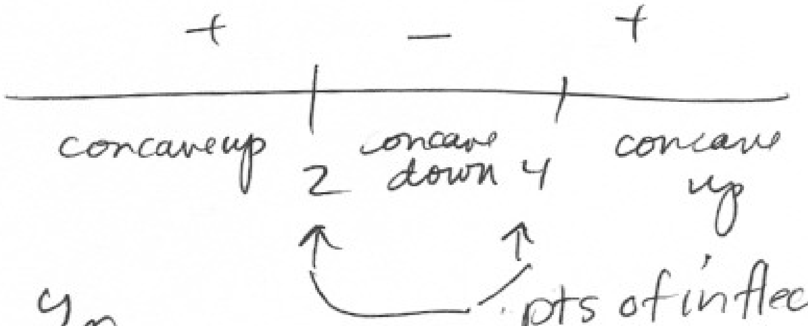
$$= -\frac{(12)(-3)}{5} = \boxed{\frac{36}{5}\text{ ft/sec}}$$

17. f cont. on $[0,8]$

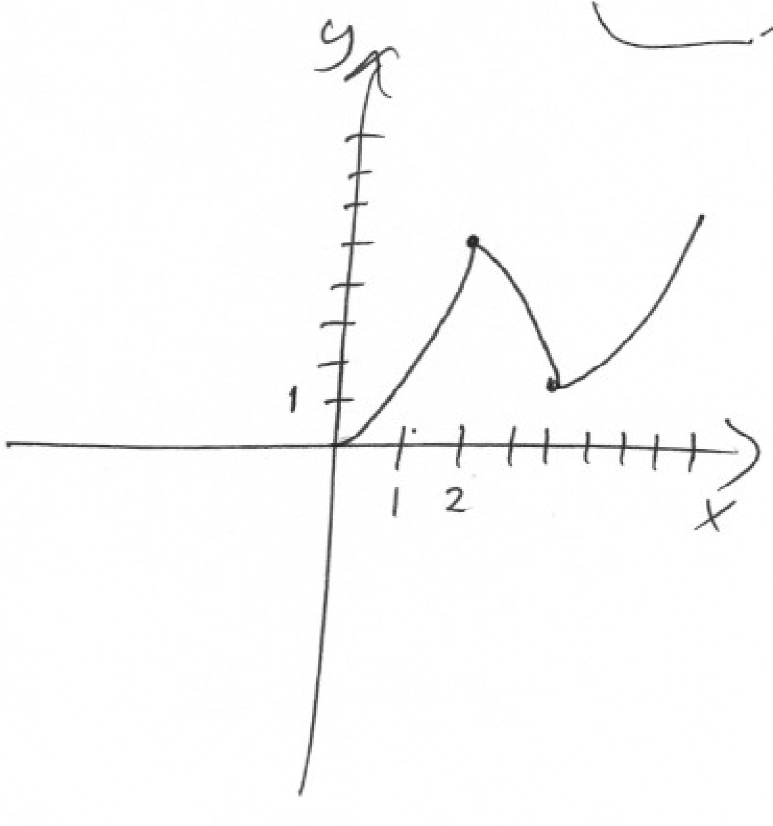
sign of $f'(x)$



sign of $f''(x)$



pts of inflection at $x=2$, $x=4$ because concavity changes.



⑱ $y = \sin(\sin x)$ $\quad x = \pi$

pt $(\pi, f(\pi)) = (\pi, \sin(\sin(\pi))$

$= (\pi, \sin(0))$

$= \underline{(\pi, 0)}$

slope $f'(\pi)$

$y' = \cos(\sin x)(\cos x)$ $\quad \boxed{\text{Chainrule}}$

$$y'\Big|_{x=\pi} = \cos(\sin \pi)\cos \pi = (1)(-1) = -1$$

Line $\quad y - 0 = -1(x - \pi)$

$$\boxed{y = -x + \pi}$$

⑲ $y = y(x)$

$xy^3 + xy = 6$ and $y(3) = 1$ find $y'(3)$

use implicit diff.

$$\frac{d}{dx}\left(xy^3 + xy\right) = \frac{d}{dx}(6)$$

$$y^3 + 3y^2y'x + y + xy' = 0$$

$$y'(3y^2x + x) = -y^3 - y$$

$$y' = \frac{-y^3 - y}{3y^2x + x}$$

$$y'\Big|_{x=3} = \frac{-(y(3))^3 - y(3)}{3(y(3))^2(3) + 3}$$

$$= \frac{-1^2 - 1}{3(1)^2(3) + 3} = \frac{-2}{12} = \boxed{-\frac{1}{6}}$$

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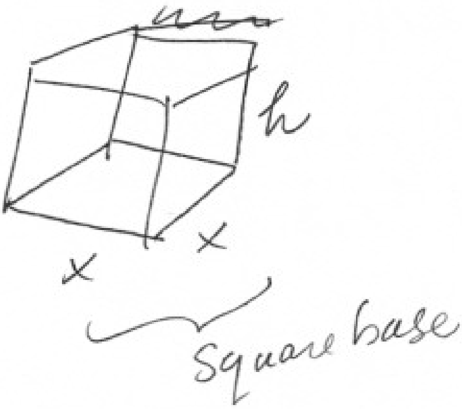

Want
$$\overset{\text{height}}{h} + \overset{\text{length}}{l} + \overset{\text{side}}{s} = 8$$

Since we have a Square base we have

$$h + 2x = 8$$

or $h = 8 - 2x$

Want
MAX VOLUME

$$V = hx^2$$

$$V = (8-2x)x^2$$

→ Need to find the Domain that is x can be no smaller than zero and no bigger than 4

(Note that when $x = 4 \Rightarrow h = 0$)

So interval is $[0,4]$

$$V = 8x^2 - 2x^3$$

$$\frac{dV}{dx} = 16x - 6x^2$$

$$= x(16 - 6x)$$

$x = 0$ or $16 = 6x$

$$x = \frac{8}{3}$$

edl C.N. + endpts

$V(0) = 0$ $V(4) = 0$

$V\left(\frac{8}{3}\right) = \boxed{18.96}$ max. volume