MATH 2425 **FALL 2003**

FINAL EXAM – VERSION A

INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 5 points each, for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. Find a formula for the *n*th partial sum of the series and use it to compute the series' sum if it converges.

$$\frac{6}{1\cdot 2}+\frac{6}{2\cdot 3}+\frac{6}{3\cdot 4}+\cdots+\frac{6}{n(n+1)}+\cdots=$$

(A) $\frac{7}{9}$ (B) $\frac{6(n+2)}{(n+1)};6$ (C) $\frac{6(n+1)}{n+2};6$ (D) $\frac{6n}{n+1};6$

(E) none of these

2. Find the sum as a function of *x*: $\sum_{k=0}^{\infty}\left(\frac{x-9}{10}\right)^k$.

(A) $\frac{10}{x+9}$ (B) $\frac{-10}{x-19}$ (C) $\frac{-10}{x+19}$ (E) $\frac{10}{x-19}$ (E) none of these

3. Find the Taylor polynomial of order 3 generated by $f(x)=\ln(x+1)$ at $a=4$.

(A) $\ln 5+\frac{x-4}{3}+\frac{(x-4)^2}{18}+\frac{(x-4)^3}{81}$ (B) $\ln 3-\frac{x-4}{3}+\frac{(x-4)^2}{18}-\frac{(x-4)^3}{81}$

(C) $\ln 5+\frac{x-4}{5}-\frac{(x-4)^2}{50}+\frac{(x-4)^3}{375}$ (D) $\ln 5-\frac{x-4}{5}+\frac{(x-4)^2}{50}-\frac{(x-4)^3}{375}$

(E) none of these

4. Which of the following represent volumes of solids of revolution revolved about the *x*-axis?

I. $\pi\int_a^b\left[f(x)\right]^2dx$ II. $2\pi\int_a^b\left[f(x)-g(x)\right]dx$ III. $\pi\int_a^b\left\{\left[f(x)\right]^2-\left[g(x)\right]^2\right\}dx$

(A) II only (B) I and II (C) I and III (D) II and III (E) I, II, and III

5. For what value of *x* does the series $\sum_{k=1}^{\infty}\frac{(x-3)^k}{k}$ converge conditionally?

(A) $x=2$ (B) $x=4$ (C) $x=1$ (D) $x=-1$ (E) $x=3$

6. Find a unit vector parallel to $\overrightarrow{PQ}$ where $P=(0,3,-2)$ and $Q=(4,2,1)$.

(A) $\langle 4,-1,3\rangle$ (B) $\frac{1}{\sqrt{18}}\langle 4,-1,-1\rangle$ (C) $\frac{1}{\sqrt{13}}\langle 0,3,-2\rangle$ (D) $\frac{1}{\sqrt{21}}\langle 4,2,1\rangle$

(E) $\frac{1}{\sqrt{26}}\langle 4,-1,3\rangle$

7. The vertices *A*, *B* and *C* of a triangle in $\mathbf{R}^3$ are given by: $A(1,1,1)$, $B(3,3,2)$, $C(3,-3,5)$ The triangle is a(n)

(A) equilateral triangle (B) right triangle (C) isosceles triangle
(D) isosceles and right triangle (E) none of these

8. The lines given by $\frac{x-4}{2}=\frac{y-6}{-3}=\frac{z+2}{5}$ and $\frac{x}{4}=\frac{y+2}{-6}=\frac{z-3}{10}$

(A) intersect at the point $(4,6,-2)$ (B) intersect at the point $(0,-2,3)$
(C) are coincident (D) are parallel (E) are skew

9. The approximate angle, in radians, between the vectors $\mathbf{u}=\langle 4,1,3\rangle$ and $\mathbf{v}=\langle -3,6,1\rangle$ is

(A) 1.658 (B) –0.0867 (C) –4.977 (D) 86.94 (E) –3

10. Find the volume of the solid obtained by rotating the region bounded by $y=x^2$, $y=0$, $x=1$, and $x=2$ about $x=1$.

(A) 2π (B) $\frac{17\pi}{6}$ (C) 3π (D) $\frac{27\pi}{6}$ (E) 5π

INSTRUCTIONS FOR PART II: For these questions, you must write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. **Draw a box around your solution.** Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

11. Compute the interval of convergence of the power series $\sum_{k=0}^{\infty}\frac{(-1)^k x^k}{\sqrt{k^2+1}}$.

12. Determine the convergence or divergence of the series $\sum_{k=0}^{\infty} ke^{-k}$.

13. Find an equation of the plane that passes through the points $(0,2,-1)$, $(1,-3,5)$, and $(3,0,-2)$.

14. Evaluate the integral $\int x(\ln x)^2\,dx$.

15. Find the value of $\int_0^e \frac{1}{x-2}\,dx$.