

## MIDTERM 2 – VERSION A

**INSTRUCTIONS FOR PART I:** Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 5 points each, for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1.  $\int_0^1 \ln x \, dx =$

A.  $\lim_{t \rightarrow 1^-} \int_0^t \ln x \, dx$

B.  $\lim_{t \rightarrow 1} \int_0^t \ln x \, dx$

C.  $\lim_{t \rightarrow 0^+} \int_t^1 \ln x \, dx$

D.  $\lim_{t \rightarrow 0} \int_t^1 \ln x \, dx$

E.  $\lim_{t \rightarrow 0^-} \int_t^1 \ln x \, dx$

2.  $\int \frac{3x^2 + 2x + 1}{x^2 + x} \, dx =$

A.  $3x + \int \frac{1+x}{x^2+x} \, dx$

B.  $\int \left( 3 + \frac{5x+1}{x^2+x} \right) dx$

C.  $\int \left( 5 + \frac{1}{x} \right) dx$

D.  $\int \left( 3 + \frac{1}{x} - \frac{2}{x+1} \right) dx$

E.  $\int \left( 3 - \frac{1}{x+1} + x \right) dx$

3. Which of the following holds for  $\int_1^{\infty} \frac{1}{x^p} \, dx$ ?

A. converges if  $p \leq 2$ , diverges if  $p > 2$

B. converges if  $p \neq 1$ , diverges if  $p = 1$

C. converges if  $p \leq 1$ , diverges if  $p > 1$

D. converges if  $p \geq 1$ , diverges if  $p < 1$

E. converges if  $p > 1$ , diverges if  $p \leq 1$

4. The sequence  $\left\{ \frac{3n-2}{n} \right\}$  is

A. increasing, bounded below but not above

B. increasing and bounded

C. decreasing, bounded above but not below

D. decreasing and bounded

E. not monotonic

5. Find the area enclosed by the graph of  $r = \theta$  for  $\theta = 0$  to  $\theta = \frac{\pi}{2}$ .

- A.  $\frac{1}{48}\pi^3$     B.  $\frac{1}{27}\pi^2$     C.  $\frac{1}{16}\pi^3$     D.  $\frac{1}{16}\pi^2$     E.  $\frac{1}{54}\pi^2$

6.  $\int \frac{1}{x^2 + 2x} dx$

I. Complete the square in the denominator to  $\int \frac{1}{(x+1)^2 - 1} dx$  and use the trig substitution  $x+1 = \sec \theta$  to obtain the correct answer.

II. Complete the square in the denominator to  $\int \frac{1}{(x+1)^2 - 1} dx$  and use the trig substitution  $x+1 = \tan \theta$  to obtain the correct answer.

III. Use partial fractions with first guess  $\frac{A}{x} + \frac{B}{x+2}$  to obtain the correct answer.

- A. I and III valid, II not valid  
B. II valid, I and III not valid  
C. I valid, II and III not valid  
D. III valid, I and II not valid  
E. I, II, III are all valid

7. Find  $\sum_{k=0}^{\infty} \frac{b_k - 3^{-k}}{2}$  given that  $\sum_{k=0}^{\infty} b_k = 0.54$ .

- A. 1.54    B. -1.54    C. -0.48    D. 1.02    E. 0.27

8.  $\sum_{k=2}^{\infty} \frac{k^2}{(k^3 - 1)^2}$

- A. converges by Direct Comparison Test with  $\frac{1}{k^6}$   
B. converges by Direct Comparison Test with  $\frac{1}{k^4}$   
C. converges by Limit Comparison Test with  $\frac{1}{k^6}$   
D. converges by Limit Comparison Test with  $\frac{1}{k^4}$   
E. diverges

9. A ball is dropped from a height of  $A$  feet. Each time the ball drops  $h$  feet, it rebounds  $0.8h$  feet. If the ball travels a total distance of 20 feet before coming to a stop, what is  $A$ ? **Hint:** Draw a picture.

- A. 12      B.  $20/9$       C. 20      D.  $5/2$       E. 4

10. For the series  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$ , which of the following is the smallest upper bound for the error if

the estimate  $\sum_{k=1}^5 \frac{(-1)^{k+1}}{k!}$  is used?

- A.  $\frac{1}{2}$       B.  $\frac{1}{6}$       C.  $\frac{1}{24}$       D.  $\frac{1}{120}$       E.  $\frac{1}{720}$

**INSTRUCTIONS FOR PART II:** For these questions, you **must** write down **all** steps in your solutions. Write legibly and carefully label any graphs or pictures. **Draw a box around your final answer.** Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

11. Evaluate  $\int_{-2}^1 \frac{1}{x^2} dx$

12. Determine if the series  $\sum_{k=0}^{\infty} \frac{1}{5} 2^{k+1} 3^{-k}$  converges or diverges. If it converges, find its value (or sum).

13. Determine if  $\sum_{k=1}^{\infty} \frac{k+1}{k^2}$  converges or diverges. Justify your answer.

14a. Does the series  $\sum_{k=1}^{\infty} \frac{e^k}{k!}$  converge?

14b. (Continuing) What is the limit of  $\left\{ \frac{e^n}{n!} \right\}$ ? Justify your answer.

15. Determine if the series  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2(2k-1)}$  converges conditionally, converges absolutely, or diverges. Justify your answer.