Math 4322

PRACTICE EXAM I

1. Compute the following:
(a) (3+4i)+(2-5i)
(b) (2+3i)(1-4i)

2. Find the real and imaginary parts of $\dfrac{\overline{(1+2i)^2}}{3-4i}$.

3. Find $\operatorname{Arg}\left(\dfrac{i}{-2-2i}\right)$.

4. Simplify $(-1+i)^{100}$ and express answer in rectangular coordinates.

5. Find $\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{\frac{1}{2}}$ and express answer in rectangular coordinates.

6. Sketch $\left\{ z \;\middle|\; \operatorname{Re}\dfrac{z-(1+i)}{z} > 0 \right\}$.

7. If $f(z) = \operatorname{Re} z$, show that f does not have a derivative for any $z \in \mathbb{C}$.

8. If $f(x+iy) = e^{-y}\cos x + ie^{-y}\sin x$, does f have a derivative? Explain your answer and if f does have a derivative, compute it.

9. Show that the function $e^y e^{ix}$ is nowhere analytic.

10. Let $u(x,y) = x^3 - 3xy^2$.
(a) Show that u is harmonic on the plane.
(b) Find all analytic $f(z)$ such that $\operatorname{Re} f(z) = u(x,y)$.

11. Find all the values of z such that $e^z = -2i$.

12. Sketch the image of $\left\{ z = x + i\dfrac{\pi}{4} \;\middle|\; x \in \mathbb{R} \right\}$ under the transformation $w = e^z$.

13. Find all values of z such that $\sin z = 2$.

14. Sketch the set of all z such that $\dfrac{\operatorname{Log}(z-1)}{z^2+1}$ is analytic.

15. Find the principle value of i^i.