

PRACTICE EXAM II

1. Evaluate $\int_C \bar{z} dz$, where C is the line segment from 0 to $1 + i$.
2. Evaluate $\int_C z^3 dz$, where C is a contour from 1 to 3.
3. a) Evaluate $\int_C \frac{1}{z} dz$, where C is a circle, centered at the origin, of radius 1, with positive orientation.
b) Evaluate $\int_C \frac{1}{z} dz$, where C is a circle, centered at the origin, of radius 2, with positive orientation.
4. Evaluate $\int_C \frac{e^z}{(z-\pi i)^2} dz$, where C is a circle, centered at the origin, radius 4 with positive orientation.
5. Find all functions $f(z)$ that satisfy all of the following conditions:
 - (1) $f(z)$ is analytic in $D = \{z \mid |z| \leq 1\}$;
 - (2) $|f(z)| \leq 3$ in D ;
 - (3) $f(0) = -3$.
6. Find the Maclaurin series of $f(z) = \frac{1}{3 - z^2}$ and its radius of convergence.
7. Find the Laurent series of $f(z) = \frac{1}{(z-1)(z-2)}$ in the annulus $1 < |z| < 2$.
8. Find $\text{Res}_{z=1} \frac{\sin z}{(z^2 - 1)(z - 2)}$.
9. Evaluate $\int_C \frac{3z^2 + 2}{(z-1)(z-2)^2} dz$, where C is a circle, centered at the origin, of radius 3, with positive orientation.