Math 4322

# PRACTICE FINAL EXAM

1. Compute $\sqrt[3]{\dfrac{2+4i}{2-i}}$.

2. Find $\operatorname{Arg}((1+3i)(2-i))$.

3. Sketch the set $\left\{ z \;\middle|\; \operatorname{Im}\dfrac{z+2i}{z} > 1 \right\}$. Is this set open or closed?

4. Does the function $f(x+iy) = x^2 - y^2 + 2xyi$ have a derivative? Explain your answer and if $f(x+iy)$ does have a derivative, compute it.

5. Show that the function $f(x+iy) = \dfrac{y+xi}{x^2+y^2}$ is nowhere analytic.

6. Find the principle value of $\log 2 - 2i$.

7. Evaluate $\int_C z^3 dz$, where $C$ is a contour made up of a line segment from 0 to 1 and an arc of a circle of radius 2 running from 1 to $i$.

8. Evaluate $\int_C \sin(z^9 + \exp z - 12)dz$, where $C$ is a circle, centered at the origin, of radius 4, with positive orientation.

9. Evaluate $\int_C \dfrac{\cos z}{(z-\pi)^3} dz$, where $C$ is a circle, centered at the origin, of radius 10, with positive orientation.

10. Prove that the function $f(z) = \begin{cases} 2\dfrac{\cos z - 1}{z^2}, z \neq 0 \\ -1, z = 0 \end{cases}$ is entire.

11. Find the Laurent series of $f(z) = \dfrac{1}{(z-2)(z-3)}$ in the annulus $2 < |z-1| < 3$.

12. Evaluate $\displaystyle\int_C \frac{z^3+1}{(z-5)(z-1)(z^2-1)} dz$, where $C$ is a circle, centered at the origin, of radius 2, with positive orientation.

13. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$.

14. Using contour integration, compute $\displaystyle\int_0^{\infty} \frac{x \sin x}{x^2+1} dx$.

15. Show that all four zeros of the polynomial $g(z) = z^4 - 7z - 1$ lie in the disk $|z| < 2$.