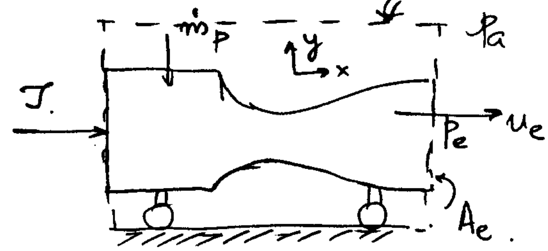
Need: T_s static thrust of rocket & turbojet engine

Given: $\dot{m}_e = 40$ kg/s Both $\frac{\dot{m}_a}{\dot{m}_f} = 50$ for turbojet

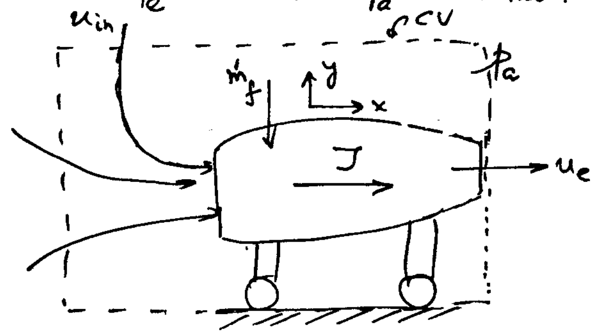
$u_e = 500$ m/s $p_e = p_a$.

Rocket: $u_e = 3000$ m/s $A_e = 0.2\,m^2$ $p_e = 0.15$ MPa $p_a = 0.101$ MPa.

Solution



Rocket



Turbojet

Assuming steady operation and using a quasi 1-D analysis, the momentum equation analysis gives finally

Rocket

$$T = \dot{m}_e u_e - \underbrace{\dot{m}_p \cancel{u_p}}_{0(2)} + (p_e - p_a) A_e .$$

$$= 3000 \frac{m}{s} \times 40 \frac{kg}{s} + (0.15 - 0.101) \times \underset{M}{10^6} \times \frac{N}{m^2} \times 0.2\,m^2$$

$$= 1.2 \times 10^5 \, N + 9.8 \times 10^3 \, N$$

$$= 1.30 \times 10^5 \, N = 130\, kN = T_s$$ (determined for stationary rocket.) ← T_{rocket}

Assumptions.

(1) Steady, quasi 1-D flow

(2) $u_p = 0$ negligible propellant x-velocity.

(3) Turbojet fuel enters at negligible x-velocity

(4) $u_{in} \approx 0$ for turbojet for CV chosen.

Turbojet.

$$T = \dot{m}_e u_e - \dot{m}_a \cancel{u_{in}} - \dot{m}_f \overset{\approx 0}{\cancel{u_f}} + \overset{0 \text{(given)}}{\cancel{(p_e - p_a)}} A_e$$

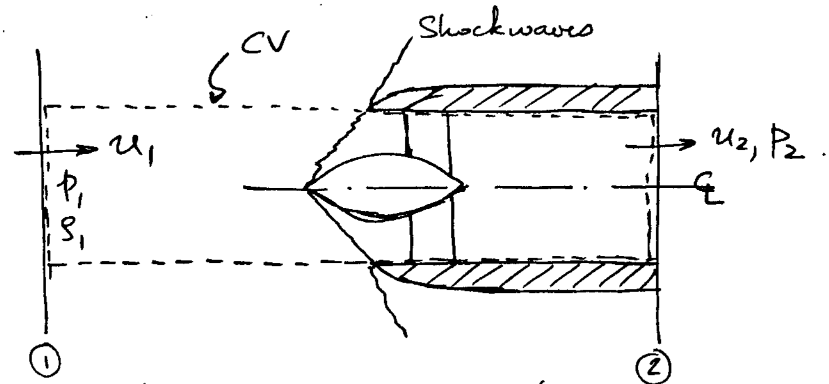
0 inlet of CV chosen far from engine (4)

$$= \dot{m}_e u_e = 40 \frac{kg}{s} \times 500 \frac{m}{s} = 2.0 \times 10^4 = 20\, kN$$ ← $T_{turbojet}$

Note: We used momentum equation <u>because</u> we had to calculate a force (thrust).

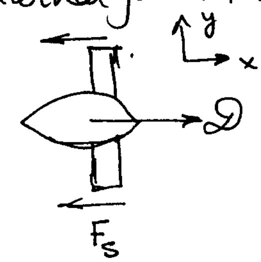
Need: Drag on centerbody and struts.

Given: Ramjet supersonic diffuser (inlet) shown on sketch – uniform flow at ① and ②



Solution

Need drag (force) hence we use a momentum analysis on CV. The drag force on centerbody and struts must be opposed by support forces in the struts where they attach to the side walls. These are the external forces that appear in the CV analysis.



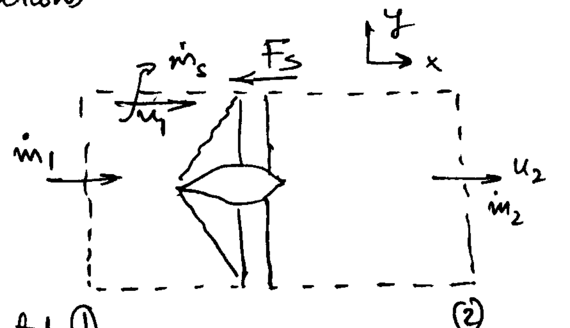
Assumptions
(1) Steady flow
(2) Quasi 1-D
(3) Shocks attached to inlet lip

$+\mathscr{D} - F_S = 0$ if struts etc don't move

$F_S = \mathscr{D}$. for assumed directions

x-momentum eqn

$$-F_S + p_1 A_1 - p_2 A_2 = \underset{1,2}{\dot{m}_2 u_2 - \dot{m}_1 u_1} + \underbrace{\cancel{\dot{m}_S} u_1}_{0\,(3)}$$



If shocks are attached to inlet lip, and since flow is undeflected ① before shocks $\dot{m}_S \equiv 0$

In steady flow $\dot{m}_1 = \dot{m}_2 = \dot{m}$ For quasi 1-D flow $\dot{m} = \rho u A$

For flow geometry specified $A_1 = A_2 = \frac{\pi D^2}{4} = A$

Rearranging and substituting $-F_S = \dot{m}(u_2 - u_1) + (P_2 - P_1)A$

$$-F_S = \frac{\pi D^2}{4}\left[\rho_1 u_1 (u_2 - u_1) + (P_2 - P_1)\right].$$

So $$\mathscr{D} = F_S = -\frac{\pi D^2}{4}\left[\rho_1 u_1 (u_2 - u_1) + (P_2 - P_1)\right].$$

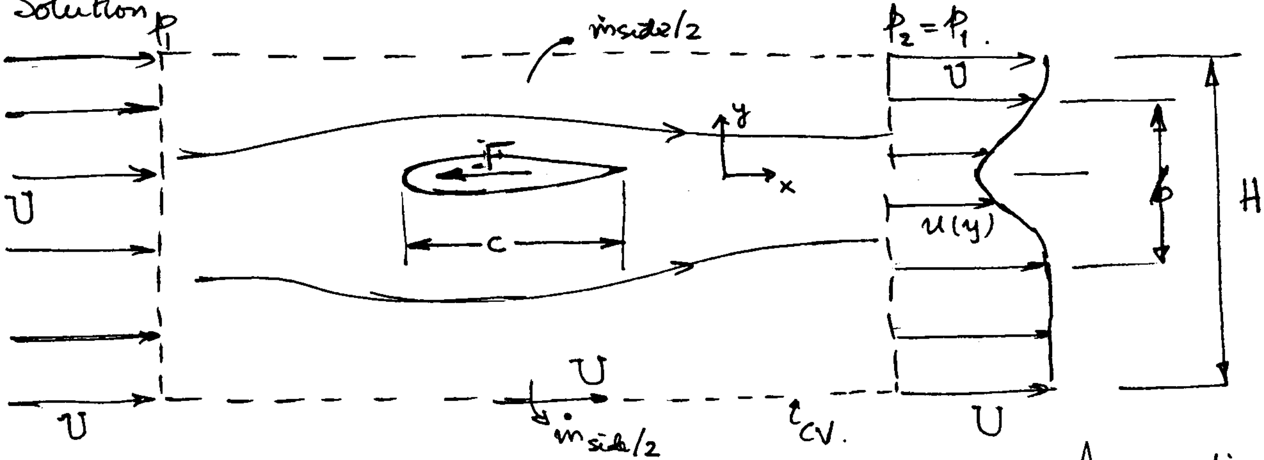
Need: $C_{\mathcal{D}}$

Given: Strut in a uniform flow generating a wake

Incompressible flow $\quad \frac{u}{U} = 1 - a\left(1 + \cos\frac{2\pi y}{b}\right) \quad -\frac{b}{2} < y < \frac{b}{2}$

$a = 0.10 \quad \frac{b}{c} = 2.0$ ← measured experimentally

Solution



$$C_{\mathcal{D}} = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 c} \quad ; \quad \mathcal{D} \text{ - drag force per unit length.}$$

Assumptions
(1) Steady flow
(2) $u = U$ on sides of CV
(3)

Need force so use momentum eq'n in x direction. Let F be the support force needed to hold strut in place, and assume depth (into page) of L

Then
$$-F + F_{px} = \underbrace{\frac{d}{dt}\int_{cv} u\rho\, d\forall}_{0\,(1)} + \int_{cs} u\rho\vec{V}\cdot d\vec{A}$$

Now $F_{px} = 0$ because streamlines are straight far downstream so there is no pressure variation across exit face and if the velocities are equal near the edges (both U) on upstream and downstream faces of the CV, then pressure is constant

Support force (F) is opposite to drag force trying to drag strut downstream in flow direction (+x, by our choice of coordinate system)

$-F = D = \mathcal{D}L$. However the <u>fluid</u> experiences a drag <u>from the strut</u> in the upstream (−ve x) direction. In a sense the support force is transmitted by the strut to the fluid.

The integral $\int_{cs} u\rho\vec{V}\cdot d\vec{A}$ has 3 contributions

$$\int_{cs} u\rho\vec{V}\cdot d\vec{A} = \int_{\text{left side}} u\rho\vec{V}\cdot d\vec{A} + \int_{\substack{\text{top}\\\text{bottom}}} u\rho\vec{V}\cdot d\vec{A} + \int_{\substack{\text{right}\\\text{side}}} u\rho\vec{V}\cdot d\vec{A}$$

$$= U\rho U HL + U\dot{m}_{side} + U\rho U(H-b)L + \rho L\int_{-b/2}^{b/2} u^2 dy$$

$$= U\dot{m}_{side} - \rho U^2 bL + \rho L\int_{-b/2}^{b/2} u^2 dy$$

Hence $\underbrace{-\mathcal{D}L}_{\text{upstream force experienced by fluid due to strut}} = U\dot{m}_{side} - \rho U^2 bL + \rho L\int_{-b/2}^{+b/2} u^2 dy \longrightarrow$ (A)

-ve x

Use mass conservation to get $\dot{m}_{side}$

$$0 = \underbrace{\cancel{\frac{d}{dt}}}_{o(1)}\int_{cv}\rho\, d\forall + \int_{cs}\rho\vec{V}\cdot d\vec{A}$$

$$\Rightarrow\quad -\rho UHL + \dot{m}_{side} + \rho U(H-b)L + \rho L\int_{-b/2}^{b/2} u\, dy = 0.$$

$$\dot{m}_{side} = \rho UbL - \rho L\int_{-b/2}^{b/2} u\, dy. \longrightarrow \text{(B)}$$

Substituting (B) into (A)

$$-\mathcal{D}L = U\left[\rho UbL - \rho L\int_{-b/2}^{b/2} u\, dy\right] - \rho U^2 bL + \rho L\int_{-b/2}^{b/2} u^2 dy.$$

or $$-\mathcal{D}\cancel{L} = \rho\cancel{L}\int_{-b/2}^{b/2} u^2 dy - \rho\cancel{L}U\int_{-b/2}^{b/2} u\, dy.$$

so $$\frac{-\mathcal{D}}{\frac{1}{2}\rho U^2 c} = \frac{2}{c}\left[\int_{-b/2}^{b/2}\frac{u^2}{U^2}dy - \int_{-b/2}^{b/2}\frac{u}{U}dy\right]$$

$$C_{\mathcal{D}} \equiv \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 c} = \frac{2}{c}\left[\int_{-b/2}^{b/2}\left(\frac{u}{U} - \frac{u^2}{U^2}\right)dy\right] = \frac{2}{c}\left[\int_{-b/2}^{+b/2}\frac{u}{U}\left(1-\frac{u}{U}\right)dy\right]$$

$$\frac{u}{U} = 1 - a\left(1 + \cos\frac{2\pi y}{b}\right).$$

$$1 - \frac{u}{U} = a\left(1 + \cos\frac{2\pi y}{b}\right).$$

$$\frac{u}{U}\left(1 - \frac{u}{U}\right) = \left[1 - a\left(1 + \cos\frac{2\pi y}{b}\right)\right] a\left(1 + \cos\frac{2\pi y}{b}\right).$$

$$= a.\left[1 + \cos\frac{2\pi y}{b} - a\left(1 + \cos\frac{2\pi y}{b}\right)^2\right]$$

$$= a\left[1 + \cos\frac{2\pi y}{b} - a\left(1 + 2\cos\frac{2\pi y}{b} + \cos^2\frac{2\pi y}{b}\right)\right]$$

$$= a\left[(1-a) + (1-2a)\cos\frac{2\pi y}{b} - a\cos^2\frac{2\pi y}{b}\right].$$

$$= a(1-a) + a(1-2a)\cos\frac{2\pi y}{b} - \frac{a^2}{2}\left(\cos\frac{4\pi y}{b} + 1\right)$$

$$= a\left(1 - \frac{3a}{2}\right) + a(1-2a)\cos\frac{2\pi y}{b} - \frac{a^2}{2}\cos\frac{4\pi y}{b}.$$

$$\int_{-b/2}^{+b/2} \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = a\left(1 - \frac{3a}{2}\right)b + a(1-2a)\frac{b}{2\pi}\left(+\sin\frac{2\pi y}{b}\right)\Bigg|_{-b/2}^{+b/2}$$

$$- \frac{a^2}{2}\cdot\frac{b}{4\pi}\left(\sin\frac{4\pi y}{b}\right)\Bigg|_{-b/2}^{+b/2}.$$

$$= a\left(1 - \frac{3a}{2}\right)b + a(1-2a)\frac{b}{2\pi}\cdot(0) - \frac{a^2}{2}\frac{b}{4\pi}\cdot(0).$$

$$= ab\left(1 - \frac{3a}{2}\right).$$

Finally: $C_{D} = \frac{2}{c}\cdot ab\left(1 - \frac{3a}{2}\right) = 2a\frac{b}{c}\left(1 - \frac{3a}{2}\right).$

Substituting $a = 0.1$ $\quad \frac{b}{c} = 2$

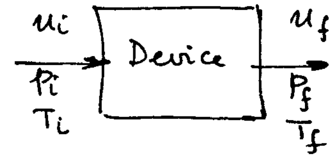
$C_{D} = 0.34$ $\quad \xleftarrow{C_{D}}$

Need: (a) w (b) $T_f = 450\,K$ possible? (c) u_{max}

Given: Perfect gas, $\hat{M} = 20$ kg/kmol $\gamma = 1.2$

Steady, adiabatic flow ; $p_i = 6$ MPa ; $T_i = 3000\,K$; $u_i = 200$ m/s

$p_f = 0.101$ MPa ; $T_f = 1800\,K$; $u_f \approx 0$



Solution:

(a) Need w $\Rightarrow$ use 1st Law (energy eq'n)

$$\cancel{q}^{\,0} + w \underset{(2)}{=} \left(h + \frac{u^2}{2}\right)_f - \left(h + \frac{u^2}{2}\right)_i$$

Steady flow form of first law needed.

Assumptions.

(1) Quasi 1-D flow

(2) Neglect grav. term

(3) Constant c_p.

$$w = (h_f - h_i) + \frac{\cancel{u_f^2}^{\,\approx 0 \text{ (given)}} - u_i^2}{2}$$

$$\underset{(3)}{=} c_p(T_f - T_i) - \frac{u_i^2}{2}$$

$$c_p = \frac{\gamma}{\gamma - 1} R = \frac{\gamma}{\gamma-1}\frac{\hat{R}}{\hat{M}} = \frac{1.2}{(1.2-1)} \times \frac{8314 \text{ J/kmol-K}}{20 \text{ kg/kmol}} = 2.49 \times 10^3 \frac{J}{kgK}$$

$$w = (-2.99 \times 10^6 - 20 \times 10^4)\frac{J}{kg} = -3.01\, 10^6 \frac{J}{kg} = -3.01 \frac{MJ}{kg} \longleftarrow w$$

$w < 0 \Rightarrow$ work done by system i.e. power transfer from device.

(b) Since the process is adiabatic, fluid entropy cannot decrease. Hence to determine if $T_f = 450\,K$ possible examine change of entropy for the process $i \to f$ $(p_i = 6\,MPa,\ T_i = 3000\,K) \to (p_f = 0.101\,MPa,\ T_f = 450\,K)$

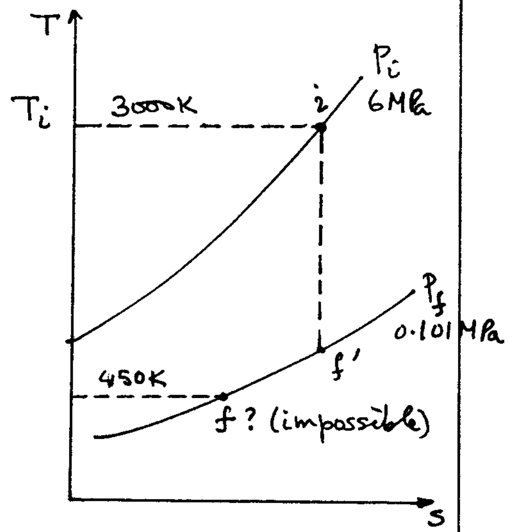
To calculate Δs use Gibb's eq'n

$$\Delta s \underset{(3)}{=} c_p \ln \frac{T_f}{T_i} - R \ln \frac{p_f}{p_i} = -4.73 \times 10^3 - (-1.70 \times 10^3) = -3.03 \times 10^3 \frac{J}{kgK}$$

$\Delta s < 0 \Rightarrow$ impossible for T_f to be as low as 450K for an adiabatic expansion. $\longleftarrow$

Alternatively one could solve for $T_{f'}$ corresponding to an isentropic expansion from i to $p_f = 0.101$ MPa. If this $T_{f'} > T_f$ specified then process is impossible.

Graphically, on a T-s diagram the question is whether the specified final state lies to the left of the initial state as shown in the sketch.



(c) $w = 0$ during expansion.

Then for an adiabatic expansion ($q = 0$)

1st law gives $h_{of} = h_{oi}$

Max velocity requires isentropic process because this gives lowest possible temperature in the final state (see sketch)

$$\frac{T_{f'}}{T_i} = \left(\frac{p_f}{p_i}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{f'} = 1520\,K$$ minimum possible final temperature.

$$h_{f'} + \frac{u_{f'}^2}{2} = h_i + \frac{u_i^2}{2}.$$

$$u_{f'}^2 = u_i^2 + 2(h_i - h_{f'}) = u_i^2 + 2c_p(T_i - T_{f'})$$

$$= 7.43 \times 10^6 \frac{m^2}{s^2}$$

$$u_{f'} = 2.73 \times 10^3\ m/s = 2.73\ km/s$$

It is convenient (and conventional) to denote the final state obtained by an isentropic process by adding a subscript s rather than using a prime as I did here. In future I will use T_{fs}, u_{fs} etc.

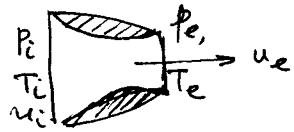
Need: u_e, p_e, a_e

Given: Jet engine nozzle at take-off. $p_i = 0.180$ MPa, $T_i = 1200$K, $u_i \approx 0$

$C_p\,[kJ/kg\text{-}K] = 0.959 + 1.16\times10^{-4}T + 3.65\times10^{-8}T^2 \qquad T[K]$

Frictionless adiabatic expansion; $\hat{M} = 30$ kg/kmol

$T_e = 1100,\ 1000,\ 900$ K



Assumptions

(1) Steady, quasi 1-D flow

(2) Ideal gas

Solution:

$$\underbrace{\cancel{q}}_{0\text{ (adiabatic given)}} + \underbrace{\cancel{w}}_{0\text{ (nozzle)}} = h_{oe} - h_{oi} \qquad \text{(1st Law)}$$

$$h_{oe} = h_e + \frac{u_e^2}{2} = h_{oi} = h_i + \underbrace{\cancel{\frac{u_i^2}{2}}}_{0\text{ (given)}}.$$

$$u_e = \sqrt{2(h_i - h_e)}.$$

$$C_{p\,(2)} = \frac{dh}{dT} = a_0 + a_1 T + a_2 T^2 \qquad a_0 = 959\ \frac{J}{kg K}$$

$$\Delta h = \int_{T_i}^{T_e} C_p\,dT \qquad a_1 = 1.16\times10^{-1}\ \frac{J}{kg K^2}$$

$$a_2 = 3.65\times10^{-5}\ \frac{J}{kg K^3}$$

$$h_e - h_i = a_0(T_e - T_i) + \frac{a_1}{2}(T_e^2 - T_i^2) + \frac{a_2}{3}(T_e^3 - T_i^3)$$

$$\Rightarrow\ h_i - h_e = a_0(T_i - T_e) + \frac{a_1}{2}(T_i^2 - T_e^2) + \frac{a_2}{3}(T_i^3 - T_e^3) \quad ; \ T_i = 1200\text{ K};\ p_i = 180\text{ kPa}$$

From a simple spreadsheet (to avoid repetitious calculations)

T_e (K)	$h_i - h_e$ (J/kg)	u_e (m/s)	p_e (kPa) see below	γ_e	a_e (m/s)	M_e
1100	1.14×10^5	478	126	1.325	635	0.75
1000	2.26×10^5	673	85.6	1.332	608	1.11
900	3.36×10^5	820	56.3	1.340	578	1.42

For an isentropic expansion $ds = 0$, so Gibbs' equation gives $\frac{dp}{p} = \frac{C_p}{R}\frac{dT}{T}$

$$\int_i^e \frac{dp}{p} = \frac{1}{R}\int_i^e \left(\frac{a_0}{T} + a_1 + a_2 T\right) dT = \frac{1}{R}\left[a_0 \ln\frac{T_e}{T_i} + a_1(T_e - T_i) + \frac{a_2}{2}(T_e^2 - T_i^2)\right]$$

$$\ln\frac{p_e}{p_i} = RHS \Rightarrow \frac{p_e}{p_i} = \exp\left[\frac{1}{R}\left\{a_0 \ln\frac{T_e}{T_i} + a_1(T_e - T_i) + \frac{a_2}{2}(T_e^2 - T_i^2)\right\}\right]$$

$R = \frac{\hat{R}}{\hat{M}} = 277 \frac{J}{kg\text{-}K}$ Values of p_e calculated from the spreadsheet are shown on Table on previous page.

$a_e = \sqrt{\gamma R T_e}$ $\gamma = \frac{c_p}{c_v} = \frac{c_p}{c_p - R}$

Values of γ, a_e and $M_e = \frac{u_e}{a_e}$ are calculated on the spreadsheet and shown on Table above.

Note: This problem has an inconsistency! For $T_e = 1100\,K$, $M_e < 1$ but $p_e = 126\,kPa$ is greater than $p_a \sim 100\,kPa$ that would be expected since problem specifies a jet engine at take-off. In reality in a subsonic flow the flow adjusts ($\dot{m}$ goes up or down as needed) so that in steady state $p_e = p_a$. Only in supersonic flow is it possible that $p_e \neq p_a$, and in supersonic flow both $p_e > p_a$ (underexpanded nozzle) and $p_e < p_a$ (overexpanded nozzle) are possible.