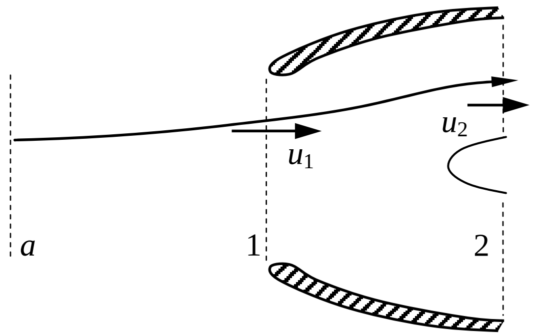
Need: A_1, A_2 for the aircraft engine inlet

Given: $\dot{m}$ = 47 kg/s, M_2 = 0.43, M_a = 0.88
p_a = 26.5 kPa, T_a = 230 K, η_d = 0.85,
$\frac{p_1 - p_a}{p_2 - p_a} \equiv \alpha = 0.45$



Solution:

Assumptions
(1) Steady, quasi-1D flow
(2) Adiabatic flow
(3) Ideal gas behavior, R_{air}=287 $J/kg\text{-}K$, γ = 1.4
(4) External flow ($a \rightarrow 1$) isentropic

Given $\dot{m}$, need A so use mass conservation equation

$$\dot{m} \underset{(1)}{=} \rho_1 A_1 u_1 \underset{(1)}{=} \rho_2 A_2 u_2$$

$$T_{o2} \underset{(2)}{=} T_{oa} = T_a\left(1+\frac{\gamma-1}{2}M_a^2\right) = 266\ K$$

$$T_2 = \frac{T_{o2}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)} = 256\ K$$

$$p_{o2} = p_a\left(1+\eta_d\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 40.9\ kPa; \qquad p_2 = \frac{p_{o2}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}} = 36.0\ kPa$$

$$\rho_2 = \frac{p_2}{RT_2} = 0.489\ kg/m^3 \qquad u_2 = M_2\sqrt{\gamma R T_2} = 0.43 \times 321\ m/s = 138\ m/s$$

Finally, $A_2 = \frac{\dot{m}}{\rho_2 u_2} = \boxed{0.696\ m^2}$

$$p_1 = p_a + \alpha\left(p_2 - p_a\right) = 30.8\ kPa;\ T_{o1} \underset{(2)}{=} T_{oa} = 266\ K;$$

A: $p_{o1} \underset{(4)}{=} p_{oa} = p_a\left(1+\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 43.9\ kPa \Rightarrow 1+\frac{\gamma-1}{2}M_1^2 = \left(\frac{p_{o1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ =1.107 $\Rightarrow M_1 = 0.730$;

$$T_1 = \frac{T_{oa}}{1+\frac{\gamma-1}{2}M_1^2} = 240\ K;\ u_1 = M_1\sqrt{\gamma R T_1} = 227\ m/s.$$

or B: $T_1 \overset{(4)}{=} T_a\left(\frac{p_1}{p_a}\right)^{\frac{\gamma-1}{\gamma}} = 240\ K;\ u_1 = \sqrt{2c_p\left(T_{o1} - T_1\right)};\ c_p = \frac{\gamma}{\gamma-1}R_{air} = 1005\ J/kg\text{-}K \Rightarrow u_1 = 227\ m/s$

After either A or B (I think procedure B is faster): we have $\rho_1 = \frac{p_1}{RT_1} = 0.447\ kg/m^3$

Finally $A_1 = \frac{\dot{m}}{\rho_1 u_1} = \boxed{0.464\ m^2}$

Need: (i) $T = f(\Delta s)$, plot of process on T-s diagram, (ii) T_2; (iii) η_c

Given: Air, compression process, $T_1 = 500\ K$, $p_1 = 200\ kPa$, $p_2 = 1\ MPa$, $\eta_{pc} = 0.92$

Solution:

Gibbs' equation: $Tds = c_p dT - \frac{1}{\rho}dp$

Assumptions
(1) Constant properties (c_p, $\gamma = 1.4$)
(2) Ideal gas, $R_{air} = 287\ J/kg\text{-}K$

For the corresponding isentropic ($ds = 0$) process with same dp:

$$0 = c_p dT_s - \frac{1}{\rho}dp \Rightarrow Tds = c_p(dT - dT_s);$$

$$\eta_{pc} \equiv \frac{dT_s}{dT} \Rightarrow dT_s = \eta_{pc} dT \text{ and } Tds = c_p dT\left(1-\eta_{pc}\right) \Rightarrow \frac{dT}{T} = \frac{ds}{c_p\left(1-\eta_{pc}\right)};$$

Integrating and rearranging one gets $\boxed{T = T_1 \exp\left[\frac{\Delta s}{c_p\left(1-\eta_{pc}\right)}\right]}$ desired expression for $T = f(\Delta s)$

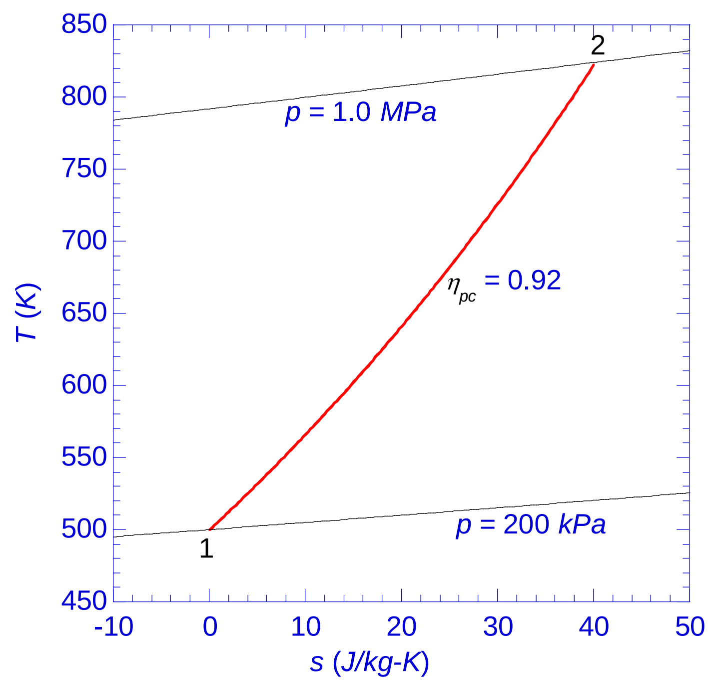
For a polytropic compression we have (from notes or equation sheet) $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\eta_{pc}\gamma}}$, so

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\eta_{pc}\gamma}} = \boxed{824\ K}$$

$$\eta_c \equiv \frac{h_{2s}-h_1}{h_2-h_1} = \frac{T_{2s}-T_1}{T_2-T_1} \qquad T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}} = 792\ K \Rightarrow \boxed{\eta_c = 0.90 < \eta_{pc}} \text{ as expected.}$$

A plot of the compression process is shown, along with constant pressure lines. For convenience, I choose state 1 as the reference state, with $s_{ref} = 0$

$$c_p = \frac{\gamma}{\gamma-1}R_{air} = 1005\ J/kg\text{-}K.$$



Need: (i) $T = f(\Delta s)$, plot of process on T-s diagram, (ii) T_2; (iii) η_t

Given: Air, expansion process, $T_1 = 1500\ K$, $p_1 = 800\ kPa$, $p_2 = 300\ kPa$, $\eta_{pe} = 0.93$

Solution:

Gibbs' equation: $Tds = c_p dT - \frac{1}{\rho}dp$

Assumptions
1. Constant properties (c_p, γ)
2. Ideal gas, $R_{air} = 287\ J/kg\text{-}K$
3. γ not specified, so use $\gamma = {}^4/_3$ appropriate for high temperature air.

For the corresponding isentropic ($ds = 0$) process with same dp:

$$0 = c_p dT_s - \frac{1}{\rho}dp \Rightarrow Tds = c_p(dT - dT_s);$$

$$\eta_{pe} \equiv \frac{dT}{dT_s} \Rightarrow dT_s = dT/\eta_{pe} \text{ and } Tds = c_p dT\left(1 - 1/\eta_{pe}\right) \Rightarrow \frac{dT}{T} = \frac{ds}{c_p\left(1-1/\eta_{pe}\right)} = -\frac{ds}{c_p\left(1/\eta_{pe}-1\right)};$$

Integrating and rearranging one gets $\boxed{T = T_1 \exp\left[-\frac{\Delta s}{c_p\left(1/\eta_{pe}-1\right)}\right]}$ desired expression for $T = f(\Delta s)$

For a polytropic expansion we have (from notes or equation sheet) $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\eta_{pe}(\gamma-1)}{\gamma}}$, so

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\eta_{pe}(\gamma-1)}{\gamma}} = \boxed{1194\ K}$$ (1156 K if $\gamma = 1.4$ assumed)

$$\eta_t \equiv \frac{h_2 - h_1}{h_{2s} - h_1} = \frac{T_2 - T_1}{T_{2s} - T_1} = \frac{T_1 - T_2}{T_1 - T_{2s}} \qquad T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}} = 1174\ K \Rightarrow \boxed{\eta_t = 0.94 > \eta_{pe}}$$ as expected.

($T_{2s} = 1133\ K$ and $\eta_t = 0.94$ for $\gamma = 1.4$)

A plot of the expansion process is shown, along with constant pressure lines. For convenience, I choose state 1 as the reference state;

$$c_p = \frac{\gamma}{\gamma - 1}R_{air} = 1148\ J/kg\text{-}K.$$