Need: (a) V_{cruise}; (b) *TSFC* (or $S_{\Im}$) at cruise; (c) Range after modification

Given: B777-200, M_e = 138,120 *kg*; M_p = 54,000 *kg*; M_f = 75,500 *kg*; M_{cruise} = 0.83 @ 39,000 *ft*
L/D = 20.5; *s* = 8400 *km*.

Solution:

From ICAO standard atmosphere at 39,000 *ft* (~11,900 *m*)
T = 216.5 *K*.

Using γ = 1.4 and R = 287 *J/kg-K*, $a = \sqrt{\gamma R T_a}$ = 295 *m/s*.

Assumptions

(1) *V*, *L/D*, *TSFC* are constant for the cruise

(2) *V*, *L/D* not changed by modification

$M_{cruise} = 0.83 \Rightarrow \boxed{V_{cruise} = 245\ m/s}$

From Breguet range equation: $S_{\Im} = \dfrac{(L/D)V}{sg}\ln\left(\dfrac{M_{TO}}{M_L}\right)$

$M_{TO} = M_e + M_p + M_f$ = 267,620 *kg*; $M_L = M_e + M_p$ = 192,120 *kg* $\Rightarrow \dfrac{M_{TO}}{M_L}$ = 1.393, so

$$S_{\Im} = \frac{20.5 \times 245 m/s}{8400 km \times 10^3 \dfrac{m}{km} \times 9.81 \dfrac{m}{s^2}} \ln(1.393) = 2.02\times10^{-5}\ s/m \Rightarrow \boxed{TSFC = 20.2\ mg/s\text{-}N}$$

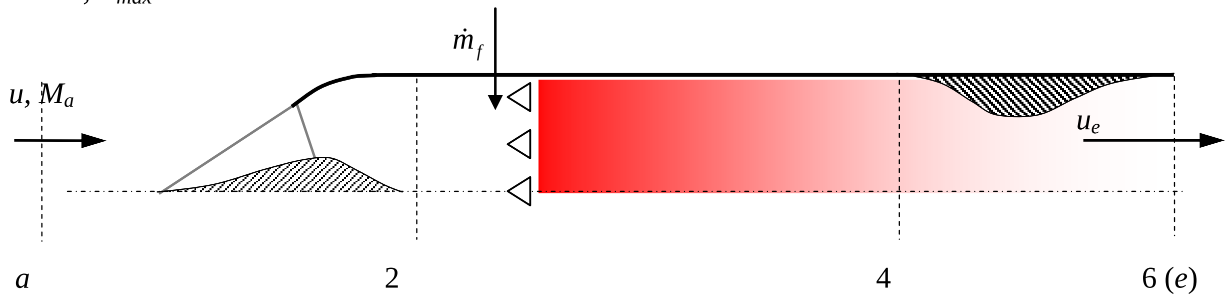
After modification $M'_e = M_e + 100kg$, no change in fuel mass

$\Rightarrow M'_{TO}$ = 267,720 *kg*, M'_L = 192,220 *kg*, $S'_{\Im} = 0.95\times 2.02\times10^{-5} = 1.92\times10^{-5}$ *mg/s-N* so range after modification is:

$$s' = \frac{V(L/D)}{S_{\Im} g}\ln\left(\frac{M'_{TO}}{M'_L}\right) = 8.84\times10^6 m \Rightarrow \boxed{s' = 8840\ km} \Rightarrow \text{Range is improved.}$$

Need: η_{th}, η_p, η_o for a ramjet

Given: $M_a = 3.5$, $T_a = 217\ K$, $|\Delta H_R| = 43\ MJ/kg$, $\eta_b = 0.99$, $r_d = 0.85$, $\eta_n = 0.96$, $r_b = 0.93$, $\gamma_h = 1.32$, $T_{max} = 2200\ K$.



Solution:

$$\Im \overset{(1)}{=} \dot{m}_a\left[(1+f)u_e - u\right] + \left(p_e - p_a\right)_{0(2)} A_e$$

$$u = M_a\sqrt{\gamma R T_a} = 1033\ m/s$$

$$T_{o2} \overset{(4)}{=} T_{oa} = T_a\left(1+\frac{\gamma-1}{2}M_a^2\right) = 749\ K$$

$$c_{ph} = \frac{\gamma_h}{\gamma_h - 1}R_{air} = 1184\frac{J}{kg-K}$$

Assumptions

(1) Steady, quasi-1D flow
(2) $p_e = p_a$ at design point
(3) Ideal gas behavior, R_{air}=287 J/kgK, piecewise constant $\gamma_c = \gamma = 1.4$, $\gamma_h = 1.32$
(4) Adiabatic components

From first law analysis applied to combustor $f = \dfrac{T_{o4} - T_{o2}}{\left[\dfrac{\eta_b|\Delta H_R|}{c_{ph}} - T_{o4}\right]}$ = 0.0430 (< $f_{max} \approx 0.067$).

$$\frac{p_{o4}}{p_e} = \underbrace{\frac{p_{o4}}{p_{o2}}}_{r_b}\underbrace{\frac{p_{o2}}{p_{oa}}}_{r_d}\frac{p_{oa}}{p_a}\underbrace{\frac{p_a}{p_e}}_{1(3)} = r_b r_d\left(1+\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 0.93\times0.85\times76.3 = 60.29$$

By assumption (4): $T_{o6} = T_{o4} = 2200\ K$; $T_{es} = T_{o4}\left(\dfrac{p_e}{p_{o4}}\right)^{\frac{\gamma_h-1}{\gamma_h}} = \dfrac{T_{o4}}{\left(\dfrac{p_{o4}}{p_e}\right)^{\frac{\gamma_h-1}{\gamma_h}}} = 814\ K$

$u_e = \sqrt{2\eta_n c_{ph}\left(T_{o4} - T_{es}\right)} = 1775\ m/s$; So $\dfrac{\Im}{\dot{m}_a} = \left[(1+f)u_e - u\right] = 818\ m/s.$

$$\eta_{th} = \frac{\dot{m}_a\left[(1+f)\dfrac{u_e^2}{2} - \dfrac{u^2}{2}\right]}{\dot{m}_f|\Delta H_R|} = \frac{\left[(1+f)\dfrac{u_e^2}{2} - \dfrac{u^2}{2}\right]}{f|\Delta H_R|}$$

= 0.600; $\boxed{\eta_{th} = 0.60}$ to within the precision of the calculations.

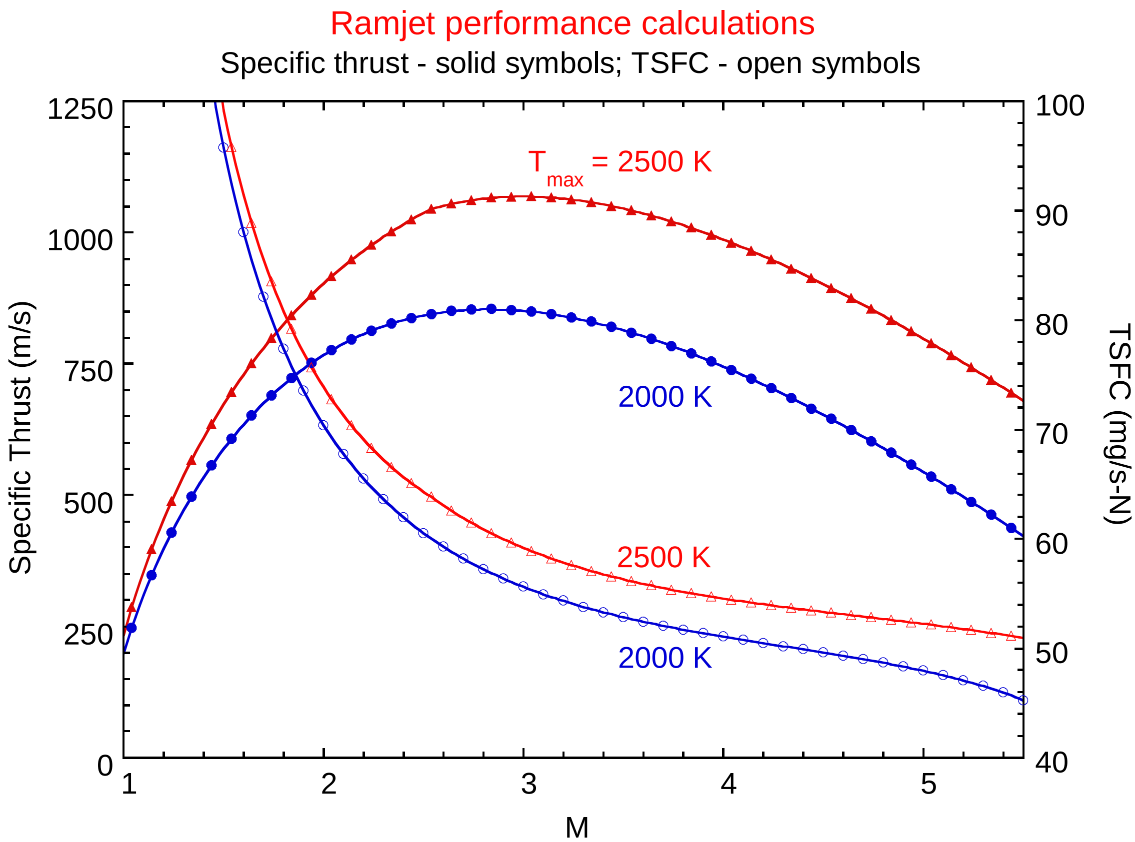
$$\eta_p = \frac{\left(\Im/\dot{m}_a\right)u}{\left[(1+f)\frac{u_e^2}{2} - \frac{u^2}{2}\right]} = \frac{\left[(1+f)u_e - u\right]u}{\left[(1+f)\frac{u_e^2}{2} - \frac{u^2}{2}\right]} = 0.762;$$ $\boxed{\eta_p = 0.76}$ to within the precision of the calculations.

$\eta_o = \eta_{th}\,\eta_p = 0.457;$ $\boxed{\eta_o = 0.46}$ to within the precision of the calculations.

Need: $\Im / \dot{m}_a$, *TSFC*, and efficiency curves as a function of *M* for a non-ideal ramjet design

Given: T_a = 220 K, $|\Delta H_R|$ = 45 MJ/kg, γ_h = 1.35, r_d = 0.88, r_b = 0.94, η_b = 0.98, r_n = 0.95

T_{max} = 2000 K and 2500 K



The variation of specific thrust and *TSFC* with *M* is shown in the figure above for T_{max} = 2000 *K*. Increasing T_{max} increases specific thrust (desirable) but also increases *TSFC* (undesirable). The peak specific thrust occurs at *M* = 2.82 for T_{max} = 2000 *K*, and at *M* = 2.99 for T_{max} = 2500 *K* showing that the maximum specific thrust shifts to higher Mach number with increase in maximum temperature in the ramjet. The curves with T_{max} = 2500 *K* are also shown on the figure for comparison. For maximum engine temperature of 2500 *K* and the Mach number range examined the fuel-air ratio is not limited, i.e. $f_{calc} < f_{stoich}$ always.

The calculations show that increasing maximum allowable engine temperature gives a larger percentage increase in specific thrust (~18% near *M* = 2 to ~47% near *M* = 5) compared to the increase in *TSFC* (~4.9% near *M* = 2 to ~9% near *M* = 5). Below *M* = 1.5 the *TSFC* is so high (>100 *mg/s-N*) that the engine is impractical. The calculations at very high *M* (> 5) are suspect as the efficiencies approach 1. It must be emphasized that the curves are for different ramjet designs – these are not the performance curves for a single ramjet as a function of flight speed. One major limitation in the analysis is the assumption of constant component stagnation pressure ratios at all flight Mach numbers. For example, the best supersonic inlet designed for *M* = 6 would really have lower stagnation pressure ratios than the best one designed for M = 2. However, the overall trends of the curves are reasonable.