Problem 5-9 Varghese.

Need: (a) Specific thrust of each of two engines

(b) $TSFC_2 / TSFC_1$

Given: Turbojet engines

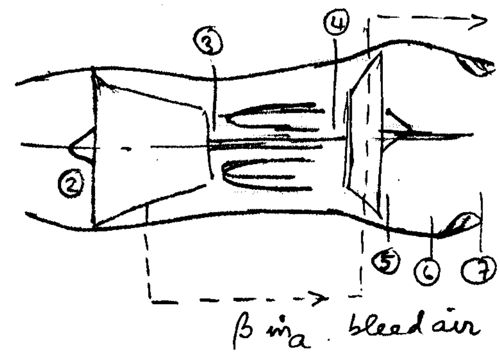
$T_{04,1} = 1200K$ $\quad T_{04,2} = 1600K$ with turbine cooling

$M_f = 2$ $\quad T_a = 200K$ $\quad \pi_c = 9$

$\gamma = 1.4$ $\quad c_p = 1.0 kJ/kg K$ constant

Ideal components

Bleed air taken where $\pi_c = 9$ ⓐ



Solution

Case 1: Standard turbojet, no bleed air

Proceeding with standard turbojet analysis we obtain the following data (summary shown)

$T_{02} = T_{0a} = 360 K$

$u = 567 m/s$

$\Delta T_{0c} = 314 K$ $\quad T_{03} = 674 K$

$f = 0.0121$

$|\Delta T_{0t}| = 314 K$ $\quad T_{06} = T_{05} = 885 K$

$\pi_t = 0.345$ $\quad \frac{p_{06}}{p_a} = 24.3$ $\quad \frac{T_7}{T_{06}} = 0.402$

Assumptions

(1) Steady, quasi 1D flow

(2) Ideal gas, const c_p

(3) $p_e = p_a$

(4) $Q_R = 45 MJ/kg$

$u_7 = 1032 m/s$

$\mathcal{T}/\dot{m}_a = 477 m/s$ $\quad TSFC_1 = 2.53 \times 10^{-5} s/m$

$(\mathcal{T}/\dot{m}_a)_1$

Case 2: 10% Bleed air: $T_{02} = T_{0a} = 360K$ as before, $u = 567 m/s$

$(\Delta T_{0c})_{main} = 314 K$ as before

$(\Delta T_{0c})_{bleed} = \frac{T_{02}}{\eta_{c,1}} \left(\pi_{c,bleed}^{\frac{\gamma-1}{\gamma}} - 1\right) = 314K$ because bleed air drawn after complete compression (9:

(not always the case)

Now $f' = \dfrac{T_{o4,2} - T_{o3}}{\frac{Q_R}{c_p} - T_{o4,2}} = \dfrac{\dot{m}_f}{\dot{m}_{a,b}}$ $= 0.0214$

$\dot{m}_{a,b} \equiv$ mass flow after bleed air removed $= (1-\beta)\,\dot{m}_a$.

Now work equation remains $|\dot{W}_T| = \dot{W}_c$.

but now $\dot{m}_t \cancel{c_p} \Delta T_{ot} = \dot{m}_a \cancel{c_p} \Delta T_{oc}$

and $\dot{m}_t = (1+f')(1-\beta)\,\dot{m}_a$

The calculations in Case 1 used $1+f \approx 1$, so to be consistent we shall use $(1+f') \approx 1$ also.

However now $|\Delta T_{ot}| = \dfrac{\Delta T_{oc}}{(1-\beta)}$ because β (bleed fraction) is not small.

Thus $|\Delta T_{ot}| = 349\,K$

so $\pi_t = \left(1 - \dfrac{\Delta T_{ot}}{\cancel{\eta_T}\, T_{o4}}\right)^{\frac{\gamma}{\gamma-1}} = 0.422$.

$\cancel{\eta_T} \to 1$

$T_{o6} = T_{o5} = T_{o4} - |\Delta T_{ot}| = 1251\,K$.

$\dfrac{p_{o6}}{p_a} = \pi_t \cdot \cancel{\pi_b}\, \pi_c \left(1 + \cancel{\eta_d} \dfrac{\gamma - 1}{2} M_f^2\right)^{\gamma/\gamma-1} = 29.73$.

$\cancel{\pi_b} \to 1$ ideal, $\cancel{\eta_d} \to 1$ ideal

$\dfrac{T_7}{T_{o6}} = \left(\dfrac{p_a}{p_{o6}}\right)^{\frac{\gamma-1}{\gamma}} = 0.379$.

$u_e = \sqrt{2 c_p T_{o6} \cancel{\eta_n} \left(1 - \dfrac{T_7}{T_{o6}}\right)} = 1249\,m/s$.

$\cancel{\eta_n} \to 1$

Now $\mathcal{T} = \dot{m}_t u_e - \dot{m}_a u = (1+f')(1-\beta)\dot{m}_a u_e - \dot{m}_a u$.

$\dfrac{\mathcal{T}}{\dot{m}_a} = (1+f')(1-\beta)\, u_e - u = 581\,m/s$. $\leftarrow (\mathcal{T}/\dot{m}_a)_2$

$TSFC_2 = \dfrac{\dot{m}_f}{\mathcal{T}} = \dfrac{f'\,\dot{m}_{ab}}{\mathcal{T}} = \dfrac{f'(1-\beta)\,\dot{m}_a}{\mathcal{T}} = \dfrac{f'(1-\beta)}{\mathcal{T}/\dot{m}_a}$

$= 3.32 \times 10^{-5}\ s/m$.

$\dfrac{TSFC_2}{TSFC_1} = 1.31$ $\leftarrow TSFC_2 / TSFC_1$

Need: Ratio of thrusts $\frac{\mathcal{T}_a}{\mathcal{T}_b}$.

Given: Turbofan engine $B = 6$

$M = 0.85$ $H = 12$ km. $\frac{p_{oe}}{p_a} = 1.6$ for both core & fan

$f = 0.018$

$T_{oe,p} = 600$ K p - primary → "core" stream

$T_{oe,f} = 340$ K f - fan stream.

$\gamma = 1.4$ $R = 287$ J/kgK. If mixed, then no drop in stagnation pressure.

Solution.

Case (a) Unmixed / separate streams.

$$\mathcal{T}_a \overset{(1)}{=} \dot{m}_a\left[(1+f)u_{ep} - u + B(u_{ef} - u)\right] + \underbrace{\cancel{(p_{ep} - p_a)}}_{o(2)}A_{ep} + \underbrace{\cancel{(p_{ef} - p_a)}}_{o(2)}A_{ef}$$

$$\frac{\mathcal{T}_a}{\dot{m}} = (\overset{(1+f)}{u_{eph}} + Bu_{ef}) - (1+B)u.$$

Assumptions.

(1) Quasi 1-D, steady flow.

(2) $p_{ep} = p_{ef} = p_a$ @ design point.

(3) $f \ll 1$ (no data to calculate it, so this is reasonable).

(4) Ideal nozzles. (given).

From Table 1 in Appendix @ 12 km altitude

$a = 295.1$ m/s so $u = M.a = 251$ m/s

For the primary nozzle. (core)

$$u_{e,p} \overset{(4)}{=} \sqrt{2c_p(T_{oe,p} - T_{e,p})} \overset{(4)}{=} \sqrt{2c_p T_{oe,p}\left[1 - \left(\frac{p_{ep}}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{2c_p T_{oe,p}\left[1 - \left(\frac{p_a}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$c_p = \frac{\gamma R}{\gamma - 1} = 1005$ J/kgK $\frac{p_a}{p_o} = \frac{1}{1.6} \Rightarrow u_{ep} = 389$ m/s.

For the secondary (fan) nozzle $u_{ef} \overset{(4)}{=} \sqrt{2c_p T_{oe,f}\left[1 - \left(\frac{p_a}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

$= 293$ m/s.

Then $\frac{\mathcal{T}_a}{\dot{m}_a} = [389(1.018) + 6 \times 293] - 7 \times 251$ m/s $= 398$ m/s.

If streams are mixed adiabatically.

$$\cancel{\dot{m}_a}(1+f)\cancel{c_p}T_{o,p} + B\cancel{\dot{m}_a}\cancel{c_p}T_{o,s} = (1+f+B)\cancel{\dot{m}_a}\cancel{c_p}T_{o,mix}$$

$$T_{o,mix} = \frac{(1+f)T_{o,p} + B\,T_{o,s}}{(1+f+B)} = 377.7\ K.$$

$$u_{e,mix} \overset{(\omega)}{=} \sqrt{2c_p T_{o,mix}\left[1-\left(\frac{P_a}{P_{o,mix}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

If there is no stagnation pressure drop when mixing then $\frac{P_a}{P_{o,mix}} = \frac{1}{1.6}$ as before.

$$u_{e,mix} = 309\ m/s.$$

$$\mathcal{T}_b = \dot{m}_a\left[(1+f+B)u_{e,mix} - (1+B)u\right] + \underbrace{(P_{e,mix} - P_a)}_{0\ @\ \text{design point}}A_e$$

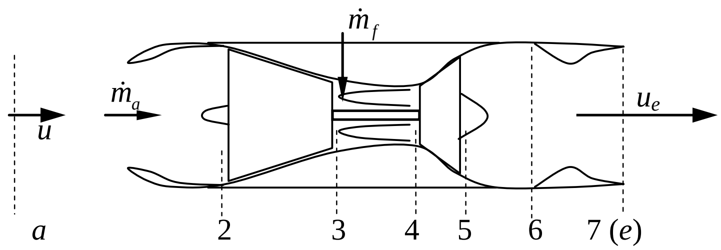
$$\frac{\mathcal{T}_b}{\dot{m}_a} = (1+f+B)u_{e,mix} - (1+B)u.$$

$$= 411\ m/s.$$

$$\frac{\mathcal{T}_a/\dot{m}_a}{\mathcal{T}_b/\dot{m}_a} = \frac{\mathcal{T}_a}{\mathcal{T}_b} = \frac{398\ m/s}{411\ m/s} = 0.968.$$

$$\frac{\mathcal{T}_{unmixed}}{\mathcal{T}_{mixed}} = 0.968 \quad \text{or} \quad \frac{\mathcal{T}_{mixed}}{\mathcal{T}_{unmixed}} = 1.033$$

Need: To explain why TSFC increases for increases in T_{o4} at fixed flight speed



Given: Turbojet engine, $\pi_c = 20$, $|\Delta H_R| = 45$ MJ/kg, $T_{max} = 1500K$, $1700\ K$, $u = 0\ m/s$ (static),
From Table 5-1:

Diffuser (d)	Compressor (c)	Burner (b)	Turbine (t)	Nozzle (n)
$\eta_d = 0.97$	$\eta_c = 0.85$	$\eta_b = 1.00$	$\eta_t = 0.90$	$\eta_n = 0.98$
$\gamma_d = 1.4$	$\gamma_c = 1.37$	$\gamma_b = 1.35$	$\gamma_t = 1.33$	$\gamma_n = 1.36$

Solution:

$$TSFC = \frac{\dot{m}_f}{\Im} = \frac{f}{\Im / \dot{m}_a}$$

For a fixed flight speed at specified altitude T_{o2} is constant and hence for fixed π_c T_{o3} is also constant. Now

$$f \overset{(1,2,3)}{=} \frac{T_{o4} - T_{o3}}{\left[\dfrac{\eta_b |\Delta H_R|}{c_{p,b}} - T_{o4}\right]}$$

Assumptions:
(1) Steady, quasi-1D flow
(2) Adiabatic components
(3) Ideal gas behavior, $R = R_{air} = 287\ J/kg\text{-}K$, piecewise constant γ for each component.
(4) $p_e = p_a$

$c_{p,b} \overset{(3)}{=} \dfrac{\gamma_b}{\gamma_b - 1} R_{air} = 1107\ J/kg\text{-}K \Rightarrow \dfrac{\eta_b |\Delta H_R|}{c_{p,b}} = 4.07\times10^4\ K >> T_{o4}$, so denominator term in f is nearly constant. Thus f grows **<u>nearly linearly</u>** with T_{o4}. (The effect of T_{o4} in the denominator is to make f increase a little *faster* than linearly with increase in T_{o4}.)

Now examine the denominator term in *TSFC*: $\dfrac{\Im}{\dot{m}_a} \overset{(1)}{=} \left[(1+f)u_e - u\right] + \dfrac{A_e}{\dot{m}_a}\underbrace{\cancel{(p_e - p_a)}}_{0(4)}$.

Because $f << 1$ typically, $(1-f) \approx$ constant, and u is fixed, we need to examine the effect of change of T_{o4} on u_e.

$$u_e = \sqrt{2\eta_n c_{p,n}\left(T_{o6} - T_{es}\right)} = \sqrt{2\eta_n c_{p,n} T_{o6}\left[1 - \left(\frac{p_e}{p_{o6}}\right)^{\frac{\gamma_n - 1}{\gamma_n}}\right]}; T_{o6} = T_{o4} - |\Delta T_{ot}|$$

For fixed π_c and flight speed ΔT_{oc} is fixed and so $|\Delta T_{ot}| = \dfrac{c_{p,c}\Delta T_{oc}}{(1+f)c_{p,t}} \approx$ constant, so T_{o6} increases linearly with T_{o4}. If we neglect the effect of increase in T_{o4} on the pressure ratio term, we see that u_e (and hence specific thrust) will increase approximately **<u>as the square root</u>** of T_{o4}. Hence we see that the numerator term in *TSFC* increases faster than the denominator (nearly linearly with T_{o4} *vs* approximately as the square root of T_{o4}). Thus *TSFC* should increase with T_{o4} at fixed flight speed.

Note: The effect of increasing T_{o4} is to increase π_t because $\pi_t = \left[1 - \frac{|\Delta T_{ot}|}{\eta_t T_{o4}}\right]^{\frac{\gamma_t}{\gamma_t - 1}}$. Now $\frac{p_e}{p_{o6}} \propto \frac{1}{\pi_t}$ so it decreases with increasing T_{o4}, and $\left[1 - \left(\frac{p_e}{p_{o6}}\right)^{\frac{\gamma_n - 1}{\gamma_n}}\right]$ increases with T_{o4}, but for typical turbojet conditions the increase is not enough to make specific thrust increase much faster than as the square root of T_{o4} and the conclusion above stands.

For the specific data in the problem statement one gets:

Assumptions (cont'd):

(5) $T_a = 288\ K$

(6) $\dot{W}_{aux} = 0$

(7) $r_b = 0.95$ (typical value)

$r_{ab} = 1$

$$c_{p,c} \overset{(3)}{=} \frac{\gamma_c}{\gamma_c - 1} R_{air} = 1063\ J/kg\text{-}K; \quad T_{o2} = T_{oa} = T_a + \frac{\cancel{u^2}_{=0}}{2c_{pc}} = 288\ K,$$

$$\Delta T_{oc} = T_{o2}\left(\frac{\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_c}\right) = 422\ K, \quad T_{o3} = T_{o2} + \Delta T_{oc} = 710\ K$$

$$c_{p,b} \overset{(3)}{=} \frac{\gamma_b}{\gamma_b - 1} R_{air} = 1084\ J/kg\text{-}K, \quad c_{p,t} \overset{(3)}{=} \frac{\gamma_t}{\gamma_t - 1} R_{air} = 1157\ J/kg\text{-}K, \quad c_{p,n} \overset{(3)}{=} \frac{\gamma_n}{\gamma_n - 1} R_{air} = 1084\ J/kg\text{-}K$$

$T_{o4,A}$ = 1500 K

$$f_A \overset{(2,3)}{=} \frac{T_{o4,A} - T_{o3}}{\left[\frac{\eta_b |\Delta H_R|}{c_{p,b}} - T_{o4,A}\right]} = 2.02\times 10^{-2}$$

$$|\Delta T_{ot,A}| \overset{(2,6)}{=} \frac{c_{p,c}}{c_{p,t}(1 + f_A)} \Delta T_{oc} = 380\ K.$$

$$T_{o5,A} = T_{o4,A} - |\Delta T_{ot,A}| = 1120\ K$$

$$\Rightarrow \pi_{t,A} = \left[1 - \frac{|\Delta T_{ot,A}|}{\eta_t T_{o4,A}}\right]^{\frac{\gamma_t}{\gamma_t - 1}} = 0.2637$$

$$\frac{p_{o6}}{p_e} = \underbrace{\cancel{\frac{p_{o6}}{p_{o5}}}}_{1(7)} \frac{p_{o5}}{p_{o4}} \underbrace{\cancel{\frac{p_{o4}}{p_{o3}}}}_{0.95(7)} \frac{p_{o3}}{p_{o2}} \underbrace{\cancel{\frac{p_{o2}}{p_a}}}_{1(M_a = 0)} \underbrace{\cancel{\frac{p_a}{p_e}}}_{1(4)} = 5.01$$

$$T_{o6,A} \overset{(2)}{=} T_{o5,A} = 1120\ K.$$

$$T_{es,A} = T_{o6,A}\left(\frac{p_e}{p_{o6}}\right)_A^{\frac{\gamma_n - 1}{\gamma_n}} = 731\ K$$

$$u_{e,A} = \sqrt{2\eta_n c_{p,n}\left(T_{o6,A} - T_{e,A}\right)} = 909\ m/s$$

$T_{o4,B}$ = 1700 K

$$f_B \overset{(2,3)}{=} \frac{T_{o4,B} - T_{o3}}{\left[\frac{\eta_b |\Delta H_R|}{c_{p,b}} - T_{o4,B}\right]} = 2.54\times 10^{-2}$$

$$|\Delta T_{ot,B}| \overset{(2,6)}{=} \frac{c_{p,c}}{c_{p,t}(1 + f_B)} \Delta T_{oc} = 378\ K.$$

$$T_{o5,B} = T_{o4,B} - |\Delta T_{ot,B}| = 1322\ K$$

$$\Rightarrow \pi_{t,b} = \left[1 - \frac{|\Delta T_{ot,B}|}{\eta_t T_{o4,B}}\right]^{\frac{\gamma_t}{\gamma_t - 1}} = 0.3184$$

$$\frac{p_{o6}}{p_e} = \underbrace{\cancel{\frac{p_{o6}}{p_{o5}}}}_{1(7)} \frac{p_{o5}}{p_{o4}} \underbrace{\cancel{\frac{p_{o4}}{p_{o3}}}}_{0.95(7)} \frac{p_{o3}}{p_{o2}} \underbrace{\cancel{\frac{p_{o2}}{p_a}}}_{1(M_a = 0)} \underbrace{\cancel{\frac{p_a}{p_e}}}_{1(4)} = 6.05$$

$$T_{o6,B} \overset{(2)}{=} T_{o5,B} = 1322\ K.$$

$$T_{es,B} = T_{o6,B}\left(\frac{p_e}{p_{o6}}\right)_B^{\frac{\gamma_n - 1}{\gamma_n}} = 821\ K$$

$$u_{e,B} = \sqrt{2\eta_n c_{p,n}\left(T_{o6,B} - T_{e,B}\right)} = 1032\ m/s$$

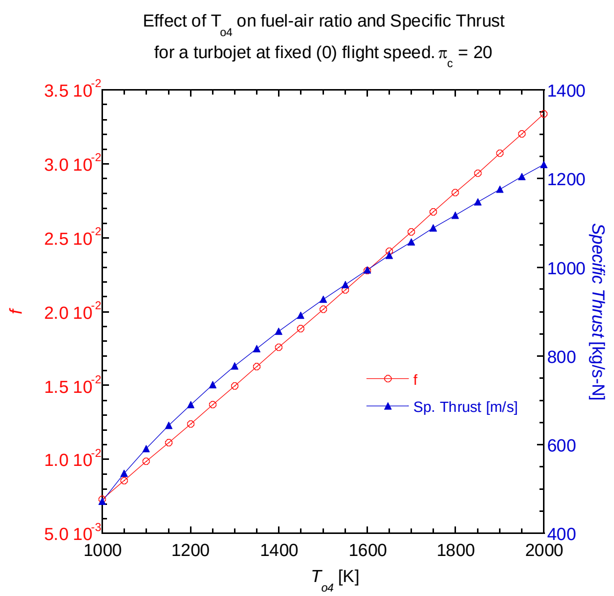
$$\left(\frac{\Im}{\dot{m}_a}\right)_A = \left[(1+f_A)u_{e,A} - u\right] = 927\ m/s \qquad \left(\frac{\Im}{\dot{m}_a}\right)_B = \left[(1+f_B)u_{e,B} - u\right] = 1058\ m/s$$

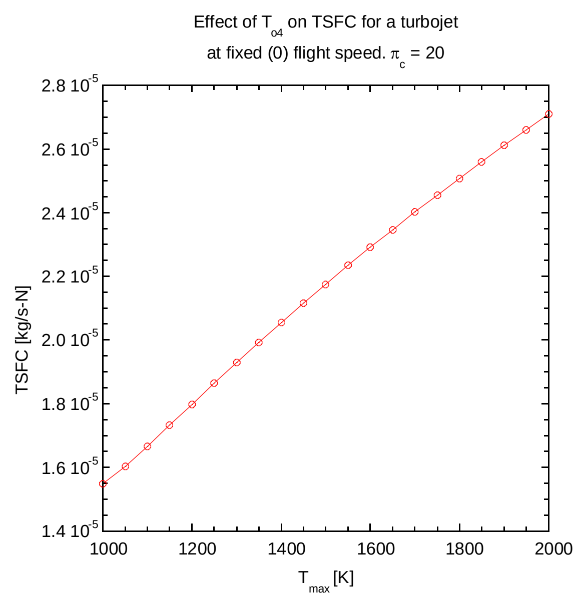
$TSFC_A = 2.18\times10^{-5}\ kg/s\text{-}N$ $\qquad$ $TSFC_A = 2.40\times10^{-5}\ kg/s\text{-}N$

As expected, the *TSFC* has increased as maximum engine temperature is increased other parameters being held fixed.

Additional comments:

(1) The figures below show the variation of f and $\Im/\dot{m}_a$ for a range of maximum engine temperatures. As expected the variation of f is nearly linear, whereas the increase of $\Im/\dot{m}_a$ is slower than linear. As a result *TSFC* increases with T_{o4}.





(2) If one does the calculations with just two values of γ, i.e. $\gamma_c = 1.4$ and $\gamma_h = 1.35$, rather than the multiple values specified in Table 5-1, then the final results are not changed significantly. Specifically, for $T_{o4} = 1500\ K$ one gets $f = 1.92\times10^{-2}$, $\Im/\dot{m}_a = 884\ m/s$, $TSFC = 2.18\times10^{-5}\ kg/s\text{-}N$.

The values of f, and $\Im/\dot{m}_a$ are each about 5% lower in the more approximate calculation, and the *TSFC* is virtually the same in this case. The 5% difference is well within what we might expect for an approximate calculation. Note that the multiple γ calculation, while more accurate, is not much better than 3-5%. The results we obtain with our simplified models give the correct ***trends,*** even though the individual values are not that accurate.

Need: $\Im/\dot m_a$, $TSFC$, η_p, η_{th}, η_{ov}

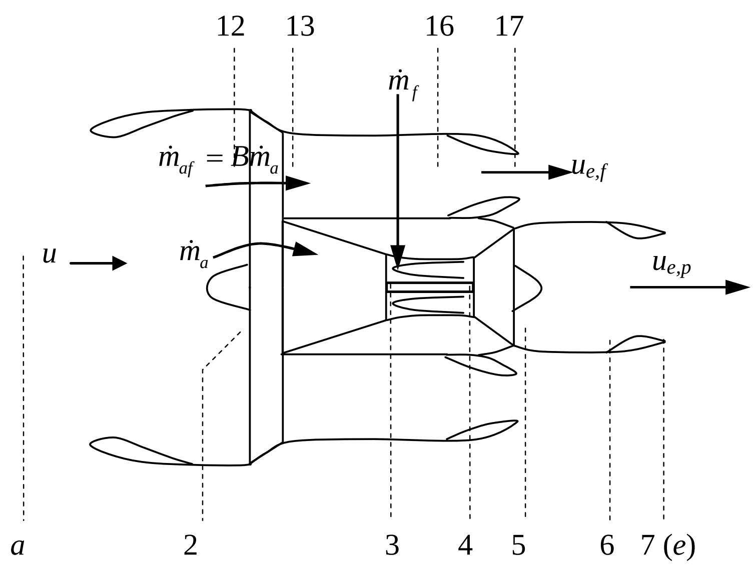
Combination of B, π_f that optimizes η_{ov}

Given: Turbofan engine, $B = 5$, $\pi_f = 1.5$, $\pi_c = 30$, $T_a = 216.7$ K, $|\Delta H_R| = 45$ MJ/kg, $T_{max} = 1700$ K

$M = 0.85$, $\eta_d = 0.97$, $\eta_c = 0.85$, $\eta_b = 1.00$, $\eta_t = 0.90$, $\eta_{n,h} = 0.98$, $\eta_f = 0.85$, $\eta_{n,c} = 0.97$

$\gamma_d = \gamma_f = 1.4$, $\gamma_c = 1.37$, $\gamma_b = 1.35$

$\gamma_t = 1.33$, $\gamma_n = 1.36$



Solution: Standard turbofan analysis but with multiple values of γ

$$\Im \overset{(1)}{=} \dot m_a\left\{\left[\left(1+f_b+f_{ab}\right)u_{e,p}-u\right]+B\left[u_{e,f}-u\right]\right\}$$
$$+\cancel{\left(p_{e,p}-p_a\right)}_{0(2)}A_{e,p}+\cancel{\left(p_{e,f}-p_a\right)}_{0(2)}A_{e,f}$$

$$u = M_a\sqrt{\gamma R T_a} = 250.8\ m/s$$

$$T_{o2} = T_{oa} = T_a\left(1+\frac{\gamma-1}{2}M_a^2\right) = 248\ K,\ T_{o4} = T_{max} = 1700\ K$$

$$\Delta T_{oc} = T_{o3}-T_{o2} = \frac{T_{o3s}-T_{o2}}{\eta_c} = \frac{T_{o2}\left(\dfrac{T_{o3s}}{T_{o2}}-1\right)}{\eta_c} \overset{(3)}{=} T_{o2}\left(\frac{\pi_c^{\frac{\gamma_c-1}{\gamma_c}}-1}{\eta_c}\right) = 439\ K$$

$$T_{o3} = T_{o2}+\Delta T_{oc} = 687\ K$$

Assumptions:

(1) Steady, quasi-1D flow

(2) $p_e = p_a$

(3) Ideal gas behavior, R_{air}=287 J/kgK, piecewise constant γ for each component

(4) Turbine exit = nozzle inlet, i.e. 5=6, no stagnation pressure drop

(5) Adiabatic components

Because of multiple γ need multiple c_p:

$$c_{pc} = \frac{\gamma_c}{\gamma_c-1}R_{air} = 1063\ J/kg\text{-}K;\ c_{pb} = \frac{\gamma_b}{\gamma_b-1}R_{air} = 1107\ J/kg\text{-}K;\ c_{pf} = \frac{\gamma}{\gamma-1}R_{air} = 1005\ J/kg\text{-}K;$$

$$c_{pt} = \frac{\gamma_t}{\gamma_t-1}R_{air} = 1157\ J/kg\text{-}K;\ c_{pn} = \frac{\gamma_n}{\gamma_n-1}R_{air} = 1084\ J/kg\text{-}K$$

$$f \overset{(3,5)}{=} \frac{T_{o4}-T_{o3}}{\left[\dfrac{\eta_b\left|\Delta H_R\right|}{c_{pb}}-T_{o4}\right]} = 2.60\times10^{-2}$$

$$\left|\dot W_t\right| = \dot W_c+\dot W_f \Rightarrow (\dot m_a+\dot m_f)\left|h_{o5}-h_{o4}\right| = \dot m_a(h_{o3}-h_{o2})+B\dot m_a(h_{o13}-h_{o12})$$

Dividing by air-flow rate through primary and using assumption (3)

$$(1+f)c_{pt}\left|T_{o5}-T_{o4}\right| = c_{pc}(T_{o3}-T_{o2}) + Bc_{pf}(T_{o13}-T_{o12}) \Rightarrow \left|\Delta T_{ot}\right| = \frac{1}{c_{ph}(1+f_b)}\left[c_{pc}\Delta T_{oc} + Bc_{pf}\Delta T_{of}\right]$$

$$\Delta T_{of} = T_{o13}-T_{o12} = \frac{T_{o13s}-T_{o12}}{\eta_f} = \frac{T_{o12}\left(\dfrac{T_{o13s}}{T_{o12}}-1\right)}{\eta_f} \overset{(3)}{=} T_{o12}\left(\frac{\pi_f^{\frac{\gamma-1}{\gamma}}-1}{\eta_f}\right) = 35.8\ K$$

$$T_{o13} = T_{o12} + \Delta T_{of} = T_{o2} + \Delta T_{of} = 284\ K$$

$$\left|\Delta T_{ot}\right| = \frac{1}{c_{ph}(1+f_b)}\left[c_{pc}\Delta T_{oc} + Bc_{pf}\Delta T_{of}\right] = 545\ K. \quad T_{o5} = T_{o4} - \left|\Delta T_{ot}\right| = 1700 - 545 = 1155\ K$$

$$\pi_t = \left(\frac{T_{o5s}}{T_{o4}}\right)^{\frac{\gamma_t}{\gamma_t-1}};\ T_{o5s} = T_{o4} - \Delta T_{ots};\ \Delta T_{ots} = \frac{\Delta T_{ot}}{\eta_t} \Rightarrow \pi_t = \left[1-\frac{\left|\Delta T_{ot}\right|}{\eta_t T_{o4}}\right]^{\frac{\gamma_t}{\gamma_t-1}} = 0.1695$$

Primary nozzle: $$\frac{p_{o6}}{p_{e,p}} = \cancelto{1(4)}{\frac{p_{o6}}{p_{o5}}}\ \frac{p_{o5}}{p_{o4}}\frac{p_{o4}}{p_{o3}}\frac{p_{o3}}{p_{o2}}\frac{p_{o2}}{p_a}\ \cancelto{1(2)}{\frac{p_a}{p_{e,p}}} = \pi_t r_b \pi_c r_d\left(1+\eta_d\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 7.72$$

$$T_{o6} \overset{(5)}{=} T_{o5} = 1155\ K; \qquad T_{es,p} = \frac{T_{o6}}{\left(\dfrac{p_{o6}}{p_{e,p}}\right)^{\frac{\gamma_n-1}{\gamma_n}}} = 672\ K$$

$$u_{e,p} = \sqrt{2c_{pn}\eta_{n,p}\left(T_{o6}-T_{es,p}\right)} = 1013\ m/s$$

Fan nozzle: $$\frac{p_{o16}}{p_{e,f}} = \frac{p_{o13}}{p_{e,f}} = \frac{p_{o13}}{p_{o12}}\frac{p_{o12}}{p_a}\ \cancelto{1(2)}{\frac{p_a}{p_{ef}}} = \pi_f\left(1+\eta_d\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 2.37$$

$$T_{o16} \overset{(5)}{=} T_{o13} = 284\ K;\ T_{es,f} = T_{o16}\left(\frac{p_{e,f}}{p_{o16}}\right)^{\frac{\gamma-1}{\gamma}} = 222\ K$$

$$u_{e,f} = \sqrt{2\eta_{nf}c_{pc}\left(T_{o13}-T_{es,f}\right)} = 348\ m/s$$

$$\frac{\Im}{\dot{m}_a} = \left\{\left[(1+f_b)u_{e,p}-u\right] + B\left[u_{e,f}-u\right]\right\} = 1.27\ km/s$$

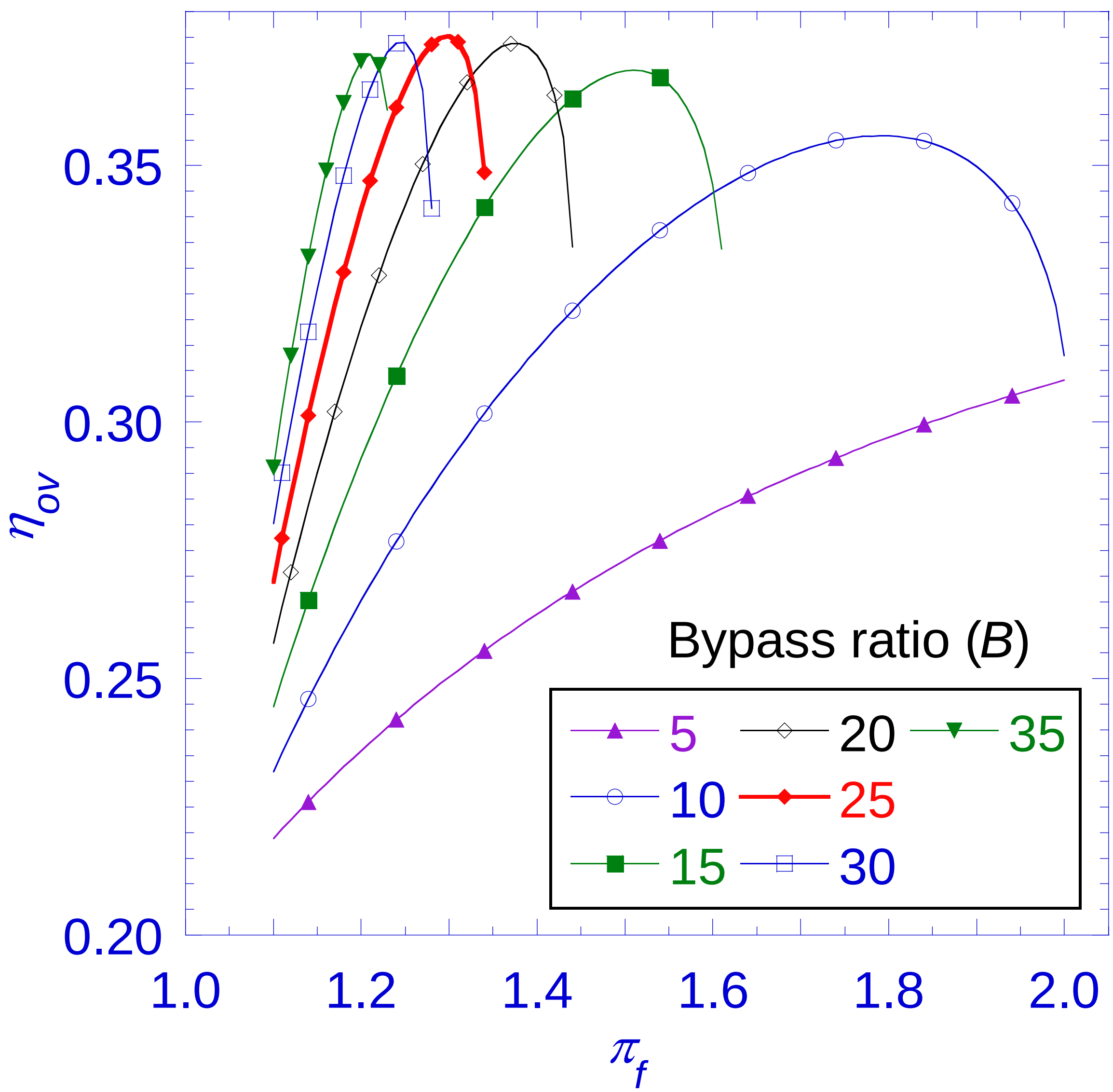
$$TSFC = \frac{\dot{m}_f}{\Im} = \frac{f}{\Im/\dot{m}_a} = 2.04\times10^{-5}\ kg/s\text{-}N$$

$$\eta_p = \frac{\left(\Im/\dot{m}_a\right)\cdot u}{\left[\left((1+f_b+f_{ab})\dfrac{u_{e,p}^2}{2}-\dfrac{u^2}{2}\right)+B\left(\dfrac{u_{e,f}^2}{2}-\dfrac{u^2}{2}\right)\right]} = 0.499$$

$$\eta_{th} = \frac{\left[\left((1+f_b)\frac{u_{e,p}^2}{2} - \frac{u^2}{2}\right) + B\left(\frac{u_{e,f}^2}{2} - \frac{u^2}{2}\right)\right]}{f\,|\Delta H_R|} = 0.547$$

$$\eta_{ov} = \eta_p \cdot \eta_{th} = 0.273$$

Note: If we used just two values of γ, γ_{cold} = 1.4, and γ_{hot} = 1.33, then we get almost exactly the same results: $\Im / \dot{m}_a$ =1.27 *km/s*, *TSFC* = 2.06×10^{-5} *kg/s-N*, η_p = 0.502, η_{th} = 0.538, and η_{ov} = 0.270. The results do not differ to within the accuracy of the calculation procedure.



The figure above shows η_{ov} as a function of π_f for a range of *B* and was calculated using a spreadsheet. Observe: (1) the optimum as a function of π_f becomes sharper as *B* increases; (2)

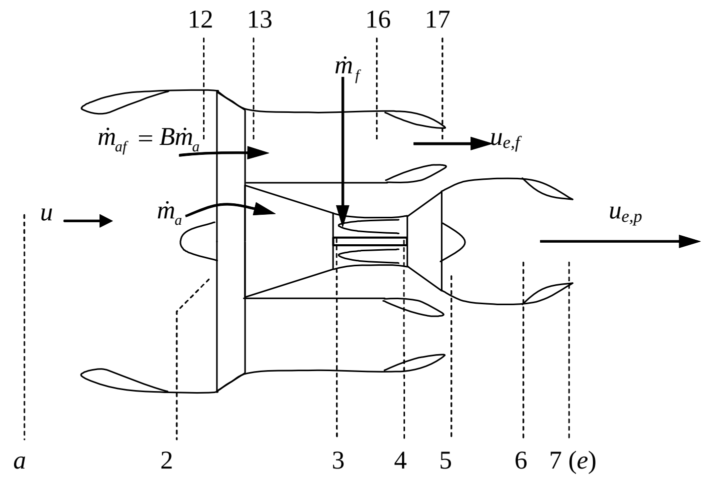
there is little further improvement beyond $B > 15$. The bypass ratio and fan pressure ratio that maximize overall efficiency are found to be near $B = 25$, $\pi_f = 1.3$, with $\eta_{ov} = 0.375$. This is an improvement of nearly 40% compared to the value for $B = 5$, $\pi_f = 1.5$. This value of bypass ratio is unrealistically high because we have assumed a constant fan efficiency independent of bypass ratio. In reality, very large fans would have substantially lower efficiency because of problems with shock losses at the fan blade tips. A more sophisticated optimization would include a model for the fan efficiency variation with bypass ratio. Additionally, the theromdynamic cycle efficiency does not account for other considerations such as the weight, drag, and noise associated with ultra-high bypass (UHB) engines. However, the above analysis illustrates why UHB ($B > 10$) engines are attractive if the technical challenges can be overcome.

Need: (a) Optimum fan pressure ratio (π_f^*) for a turbofan with separate primary (core) and secondary (fan) streams.

Given:

Flight speed Mach number, M_a	0.80	Fuel enthalpy of reaction, $\lvert\Delta H_R\rvert$	45 *MJ/kg*
Ambient temperature, T_a	217 *K*	Burner efficiency, η_b	0.99
Inlet adiabatic efficiency, η_d	0.92	Burner stagnation pressure ratio, r_b	0.95
Compressor pressure ratio, π_c	40	Turbine inlet temperature, T_{max}	1640 *K*
Compressor & fan polytropic efficiency, η_{pc}, η_{pf}	0.92	Hot gas specific heat, γ_h	1.33
Ambient pressure, p_a	19.3 *kPa*	Turbine polytropic efficiency, η_{pt}	0.93
Bypass ratio, B	8	Primary & fan nozzle efficiencies, η_n	0.97

Solution:



We assume $p_e = p_a$ for both streams, and we seek an optimum π_f for fixed u, T_a, π_c, B, T_{o4}.

Specific thrust, $S \equiv \left(\Im/\dot{m}_{tot}\right)$,

$$S = \left\{\left[\left(1+f_b\right)u_{e,p} - u\right] + B\left[u_{e,f} - u\right]\right\}\Big/\left(1+B\right)$$

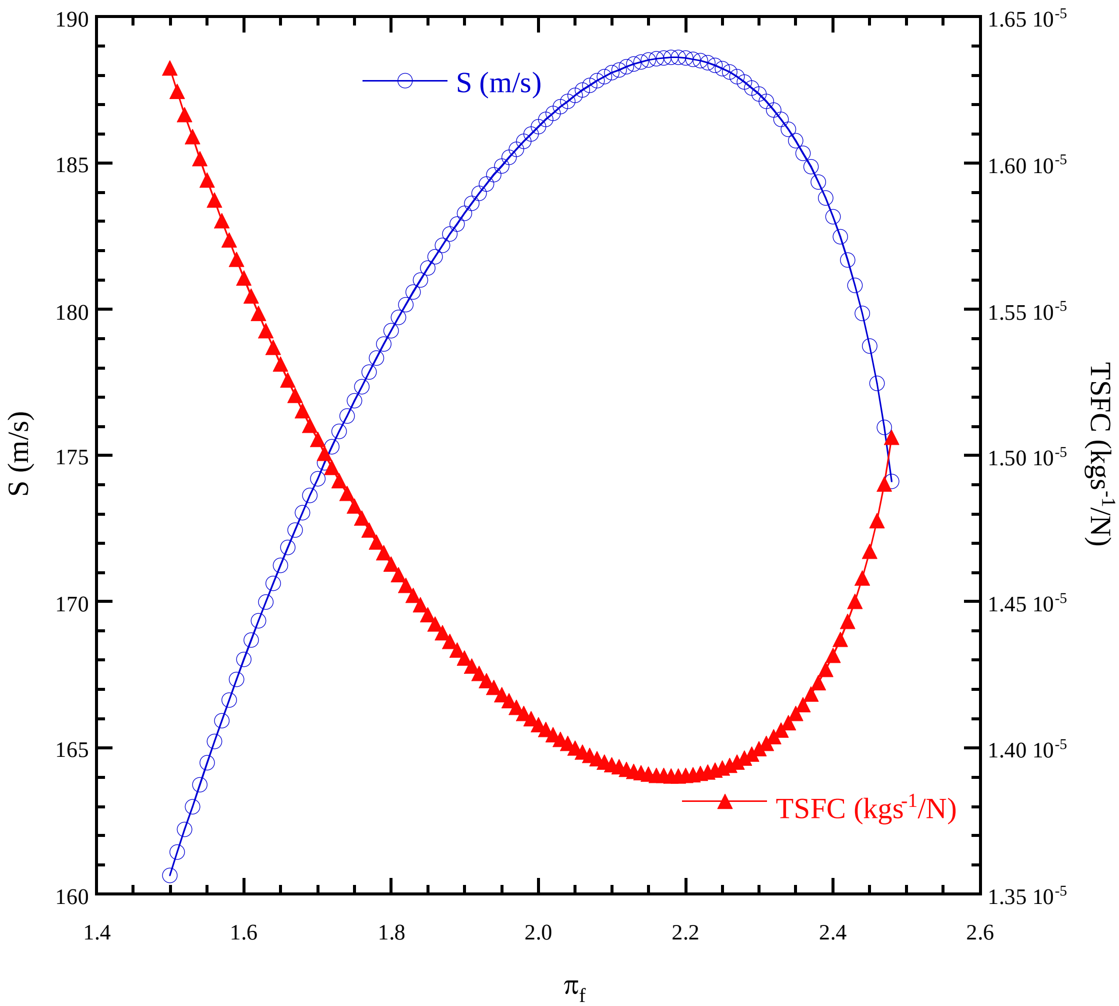
and $\dfrac{\partial S}{\partial \pi_f} = \dfrac{1}{\left(1+B\right)}\left\{\left(1+f_b\right)\dfrac{\partial u_{e,p}}{\partial \pi_f} + B\dfrac{\partial u_{e,f}}{\partial \pi_f}\right\}$.

Now $\dfrac{\partial u_{e,f}}{\partial \pi_f} > 0$, because increasing π_f increases the pressure ratio across the fan nozzle. Also $\dfrac{\partial u_{e,p}}{\partial \pi_f} < 0$ because increasing π_f increases the power that must be extracted from the turbine, and thus increases pressure drop across the turbine and hence reduces the pressure ratio across the primary nozzle. At very low π_f the positive second term is larger than the first, but as π_f grows the negative first term increases until the two are equal, we have a maximum ($\partial S/\partial \pi_f$ = 0), and further increases in π_f reduce S.

A little thought shows why the same π_f simultaneously maximizes S and minimizes $TSFC$.

$TSFC = \dfrac{\dot{m}_f}{\Im} = \dfrac{\dot{m}_f/\dot{m}_a}{\Im/\dot{m}_a} = \dfrac{f_b}{\left(\Im/\dot{m}_{tot}\right)}\dfrac{1}{\left(1+B\right)} = \dfrac{f_b}{S}\dfrac{1}{\left(1+B\right)}$. For fixed u, T_a we have constant T_{o2}, and for fixed π_c and compressor efficiency (either adiabatic or polytropic) T_{o3} is also fixed. Because T_{o4} is constant, then f_b is also constant independent of π_f. Because B is also held constant, we see immediately that maximizing S will minimize $TSFC$. Deriving an explicit analytic expression for the optimum π_f is very tedious algebraically especially for non-ideal components, but it is easy to compute S for a range of π_f using a spreadsheet and find the optimum π_f numerically. The only caution is that one cannot use too large a step (especially for high B), because if one uses too large a value of π_f, then the primary nozzle has a pressure ratio that is less than 1, which is impossible. The spreadsheet on ERes has to be modified to handle the case where polytropic efficiency of compressor, fan and turbine are specified. The equations required are on the equation sheet.

Using such a spreadsheet, for the parameters given I obtain $\pi_f^* = 2.18$ with $S_{max} = 188.6\ m/s$ and $TSFC_{min} = 1.390\times10^{-5}$ kg $s^{-1}/N = 13.90\ mg\ s^{-1}/N$. I am quoting more digits than usual because here we are comparing relative values for slightly different values of π_f. Increasing π_c to 45 decreases π_f^* to 2.16. The optimum fan pressure ratio makes the two exhaust velocities nearly equal ($u_{e,p} = 472\ m/s$, $u_{e,f} = 417.5\ m/s$). They are not exactly equal at the optimum point because of non-ideal components. Using the spreadsheet with all component efficiencies set to unity we see that the optimum fan pressure ratio (which is substantially higher in this case) gives exactly equal primary and fan exit velocities. This can be proven mathematically for this case, but the algebra is tedious.



Need: CF6 engine performance computed with NASA Engine simulator for range of conditions specified on problem sheet
Given: Data on problem sheet
Solution:
Results from NASA EngineSim.

Thrust in *kN*:

ξ=100%	*M*									
H(m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	201.2	185.6	172.3	161.1	151.8	142.8				
3000				134.6	127.4	120.3	115.6	111.9		
6000						100.0	96.3	93.4	91.2	90.7
9000						81.9	79.1	76.8	75.1	74.7
12000						60.4	58.4	56.8	55.6	55.2

TSFC in $mg\text{-}s^{-1}/N$:

ξ=100%	*M*									
H(m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	8.66	9.41	10.2	11.0	11.8	12.5				
3000				10.9	11.7	12.3	13.1	13.8		
6000						12.2	12.9	13.6	14.2	15.0
9000						12.0	12.7	13.3	13.9	14.6
12000						11.9	12.5	13.2	13.8	14.4

Thrust in *kN*:

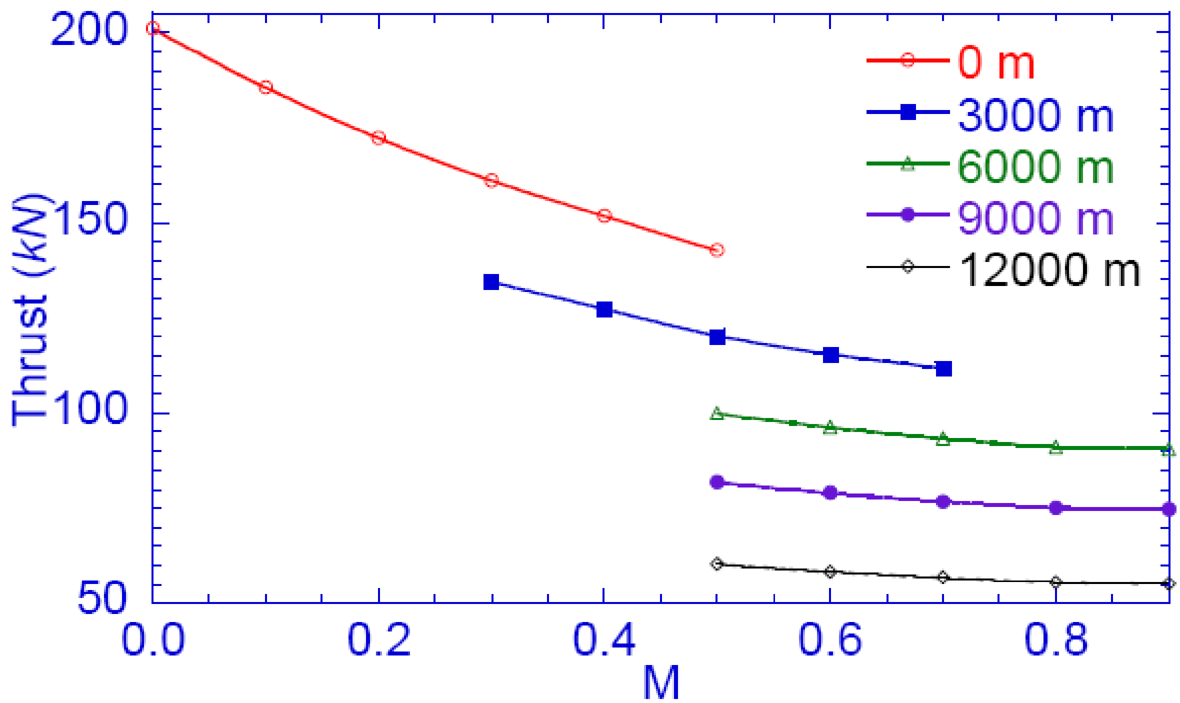
ξ=75%	*M*									
H(m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	94.3	84.7	76.7	70.0	64.4	59.8				
3000				58.7	54.4	50.9	48.0	46.0		
6000						42.5	40.4	38.8	37.7	36.8
9000						34.9	33.4	32.1	31.2	30.6
12000						25.7	24.7	23.8	23.2	22.7

TSFC in $mg\text{-}s^{-1}/N$:

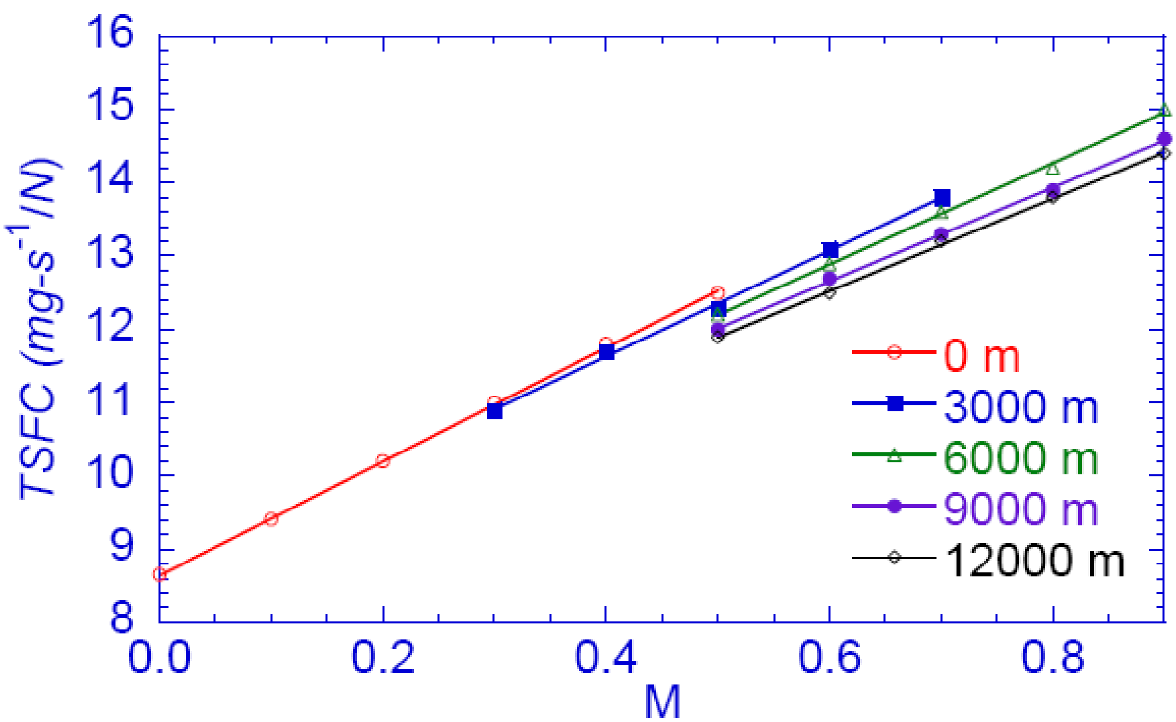
ξ=75%	*M*									
H(m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	6.88	7.67	8.51	9.41	10.3	11.3				
3000				9.35	10.2	11.1	11.9	12.7		
6000						10.9	11.7	12.4	13.1	13.7
9000						10.7	11.4	12.1	12.7	13.3
12000						10.6	11.3	11.9	12.5	13.1

Plots of $\Im$ and *TSFC* for these throttle settings are shown below.

(i)

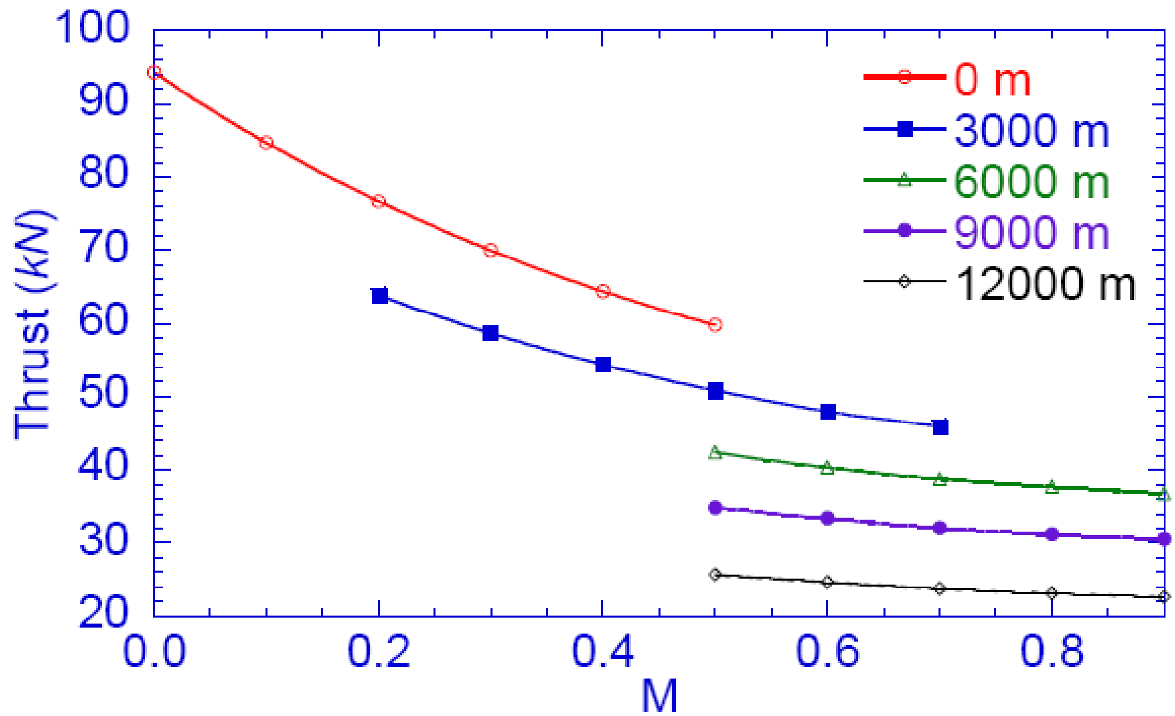


$\Im$ *vs* M for ξ = 100 %

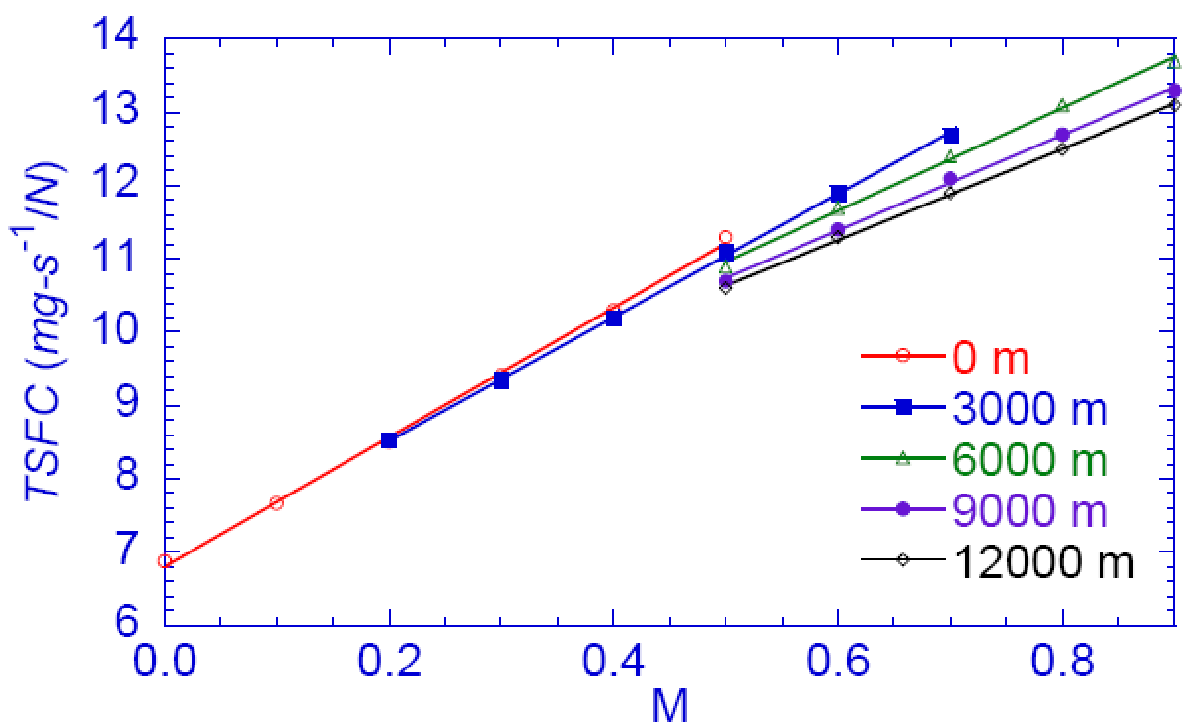


TSFC vs M for ξ = 100 %

(ii)



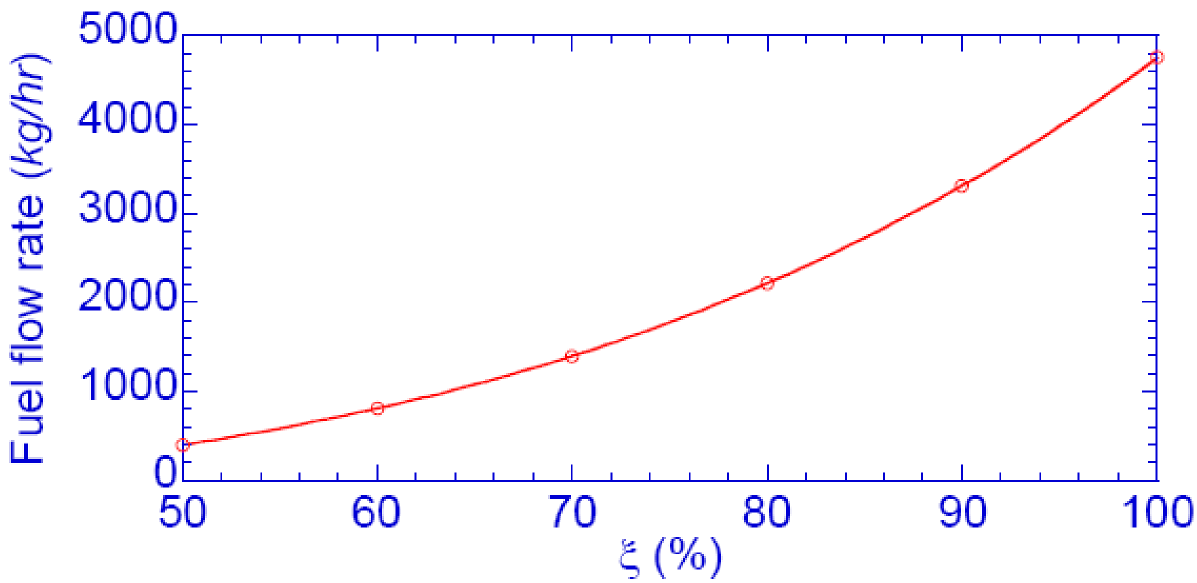
$\Im$ *vs M* for ξ = 75 %



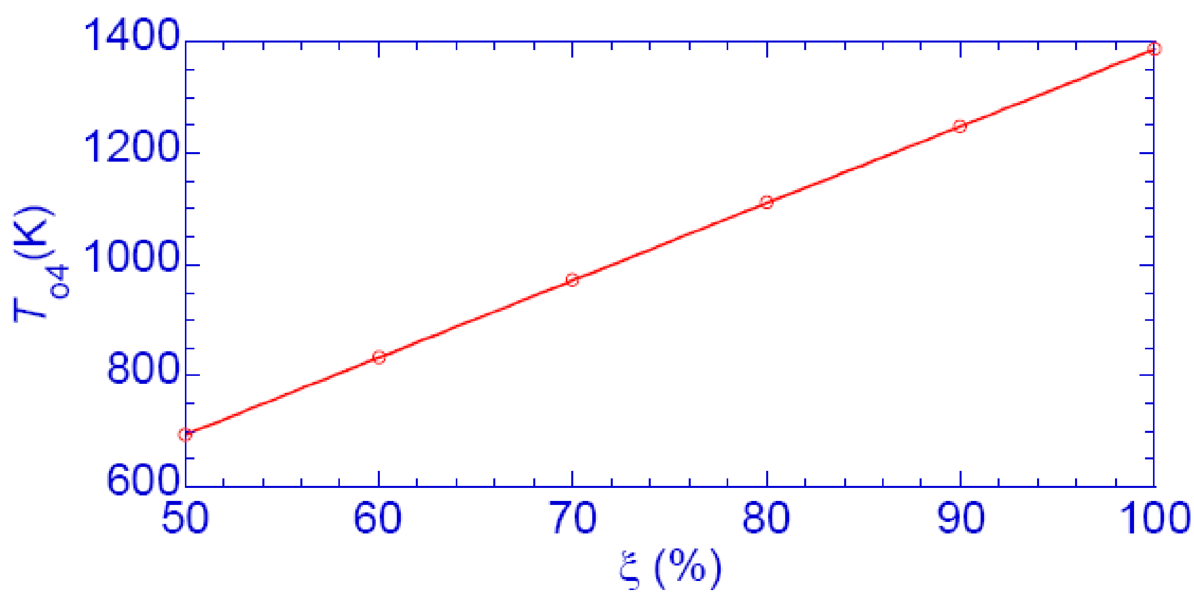
TSFC vs M for ξ = 75 %

(iii) Comparing the listed temperature and pressure for station 1 at any given flight altitude and Mach number > 0. For example, at H = 0 m, and M = 0.5, p_a = 101.3 kPa, T_a = 288 K. The corresponding stagnation values are p_{oa}= 120.1 kPa, T_{oa} = 302 K. These match the listed values of pressure and temperature at station 1, indicating that the values listed correspond to the stagnation values.

(iv) Changing throttle setting changes turbine inlet temperature and fuel flow rate as expected, i.e. decreasing the throttle setting from 100% reduces turbine inlet temperature and fuel flow rate. Changing M or H has no effect on T_{o4} in the simulation model.



Variation of fuel flow rate with throttle setting at M = 0.8, H = 6000 m.

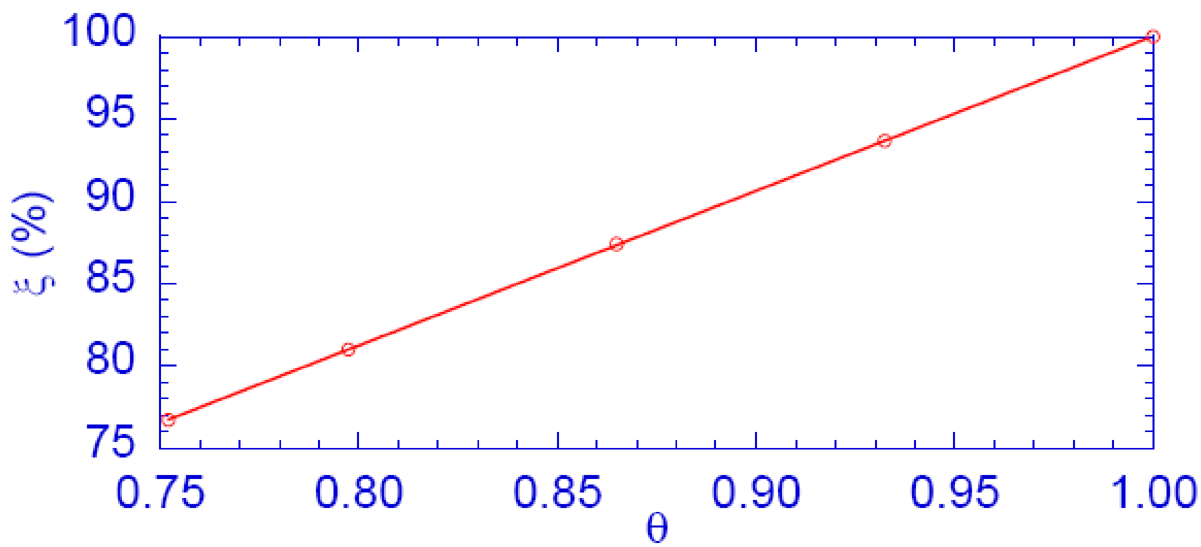


Variation of T_{o4} with throttle setting independent of M and H

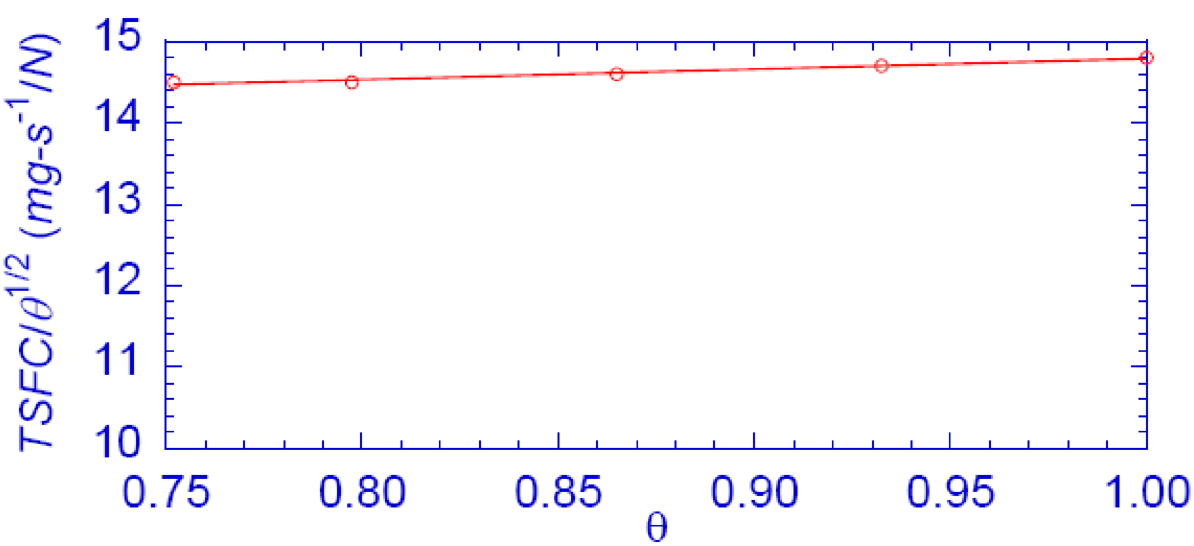
The plots show that T_{o4} varies linearly with ξ but fuel flow rate varies non-linearly with ξ.

H (m)	p_a (kPa)	T_a (K)	δ	θ	$\Im/\delta$ (kN)	$\Im$ (kN)	ξ (%)	$\dot{m}_f$ (kg/hr)	$TSFC$ (mg/s-N)	$TSFC/\sqrt{\theta}$ (mg/s-N)
0	101.3	288	1.000	1.000	128.8	128.8	100.0	6850	14.8	14.8
3000	70.1	269	0.692	0.932	128.8	89.1	93.7	4547	14.2	14.7
6000	47.2	249	0.466	0.865	128.8	60.0	87.4	2935	13.6	14.6
9000	30.8	230	0.304	0.797	128.8	39.1	81.0	1825	13.0	14.5
12000	19.3	217	0.190	0.752	128.8	24.5	76.7	1105	12.5	14.5

Looking at ξ we see that it is given by θ (expressed as a percentage) to a good approximation, whereas $TSFC/\sqrt{\theta}$ is almost constant.



Variation of ξ with θ



Variation of $TSFC/\sqrt{\theta}$ with θ