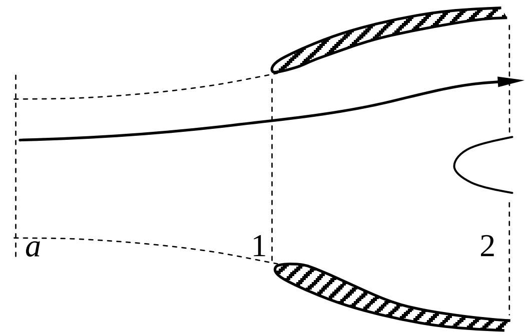
Need: (a) p_1/p_a; (b) p_2/p_1 (c) $(p_2 - p_1)/q_1$

Given: Aircraft flying at $M_a = 0.9$, $\dot{m}_a = 100\ kg/s$, $A_1 = 3.07\ m^2$, $\eta_d = 0.90$, $M_2 = 0.4$, $p_a = 9.57\ kPa$, $T_a = 222\ K$



Solution:

Assumptions
1. (1) Steady, quasi-1D flow
2. (2) Adiabatic flow
3. (3) Ideal gas behavior, $R_{air}=287\ J/kg\text{-}K$, $\gamma = 1.4$
4. (4) External flow ($a \to 1$) isentropic

$$\dot{m} = \rho_a A_a u_a = \rho_1 A_1 u_1$$

$$u_a = M_a \sqrt{\gamma R T_a} = 269\ m/s$$

$$\rho_a = \frac{p_a}{RT_a} = \frac{9.57 \times 10^3 \frac{N}{m^2}}{287 \frac{J}{kg-K} 222K} = 0.150\ kg/m^3$$

$$A_a = \frac{\dot{m}_a}{\rho_a u_a} = \frac{100\,kg/s}{0.150\,kg/m^3\ 269\,m/s} = 2.48\ m^2 < A_1$$ $\Rightarrow$ external deceleration (and compression) of the flow. Because we have a compressible flow ρ varies with M and hence one would have to solve a non-linear equation to find M_1. It is easier to use the isentropic flow tables. *In an isentropic flow* the ratio A/A^* is a unique function of M, where A^* is the area that would pass the entire flow at $M^* = 1$, and this quantity is listed in isentropic flow tables and calculated in the isentropic flow spreadsheet. By assumption (4) flow ($a \to 1$) isentropic, so

$$\frac{A_1}{A^*} = \frac{A_1}{A_a}\frac{A_a}{A^*} = \frac{A_1}{A_a}\left(\frac{A_a}{A^*}\right)_{M_a} = \frac{3.07}{2.48} \times 1.009 = 1.25$$

Isentropic flow tables give $A_a/A^* = 1.009$ at $M = 0.9$ for $\gamma = 1.40$, and $A_1/A^* = 1.25 \Rightarrow$ $M_1 = 0.553$ via interpolation in the tables.

Then, because p_o = constant in an isentropic flow then by assumption (4)

$$\frac{p_1}{p_a} = \frac{p_o/p_a}{p_o/p_1} = \left[\frac{1+\frac{\gamma-1}{2}M_a^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}} = \left[\frac{1.162}{1.061}\right]^{3.5} \Rightarrow \boxed{p_1/p_a = 1.37}$$

Using the standard definition of η_d, $\frac{p_{o2}}{p_a} = \left(1 + \eta_d \frac{\gamma-1}{2} M_a^2\right)^{\frac{\gamma}{\gamma-1}} = 1.61$

$$\frac{p_{o2}}{p_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} = 1.117;\ \frac{p_2}{p_1} = \frac{p_2}{p_{o2}}\frac{p_{o2}}{p_a}\frac{p_a}{p_1} = \frac{1}{1.117} \cdot 1.61 \cdot \frac{1}{1.37} \Rightarrow \boxed{p_2/p_1 = 1.05}$$

(The answer at the back of the book corresponds to a non-standard (incorrect?) operational definition of η_d, corresponding to $\frac{p_{o2}}{p_1} \overset{(?)}{=} \left(1 + \eta_d \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$.)

The inlet dynamic pressure $q_1 \equiv \frac{1}{2}\rho_1 u_1^2 = \frac{1}{2}\gamma p_1 M_1^2$, so $\frac{p_2 - p_1}{q_1} = \frac{(p_2/p_1) - 1}{q_1/p_1} = \frac{(p_2/p_1) - 1}{\frac{1}{2}\gamma M_1^2} \Rightarrow$

$\boxed{(p_2 - p_1)/q_1 = 0.229}$

A more standard measure of static pressure recovery in an inlet is the total static pressure rise from the *freestream* expressed as a fraction of the *freestream* dynamic pressure, i.e.

$\frac{p_2 - p_a}{q_a} = \frac{(p_2/p_a) - 1}{\frac{1}{2}\gamma M_a^2}$ (= 0.78 in this case)

Need: (1) Balanced chemical reactions for (a) stoichiometric combustion, (b) 300% excess air

(2) fuel-air mass ratio (*f*) corresponding to stoichiometric proportion

Given: Hydrocarbon jet fuel with a mean C/H ratio of 1:1.6

Solution:

Overall reaction

$$CH_{1.6} + 1.4\,(O_2 + 3.76\,N_2) \rightarrow (1)\,CO_2 + 0.8\,H_2O + 5.26\,N_2$$

Recall from elementary chemistry that for a balanced chemical reaction must have the same number of atoms of each kind in reactants and products. Check:

Atom	Reactants	Products
C	1	1
H	1.6	0.8×2=1.6
O	1.4×2=2.8	1×2+0.8×1=2.8
N	1.4×3.76×2=5.26×2	5.26×2

For 300% excess air, multiply stoichiometric proportion by 3.

$$CH_{1.6} + 4.2\,(O_2 + 3.76\,N_2) \rightarrow 1\,CO_2 + 0.8\,H_2O + 15.79\,N_2 + 2.8\,O_2$$

Major products are CO_2, H_2O and excess air. A check similar to the one above can easily be done.

Using $m_C = 12$, $m_H = 1$, $m_O = 16$, $m_N = 14$ (in atomic mass units, *amu*) we have

$m_{fuel} = 1\times12 + 1.6\times1 = 13.6$ *amu*, $m_{air} = 1.4(32+3.76\times28) = 192.2$ *amu*, so $f = 7.08\times10^{-2}$

Need: (1) Balanced chemical reaction for stoichiometric combustion

(2) fuel-air ratio (f) for combustion with $\phi = 0.85$

Given: Hydrogen fuel

Solution:

Overall reaction at stoichiometric proportions

$$H_2 + 0.5\,(O_2 + 3.76\,N_2) \rightarrow 1\,H_2O + 1.88\,N_2$$

(Although it isn't really needed to solve the problem, for equivalence ratio $\phi = 0.85$ the overall reaction could be written:

$$0.85\,H_2 + 0.5\,(O_2 + 3.76\,N_2) \rightarrow 0.85\,H_2O + 0.075\,O_2 + 1.88\,N_2)$$

Using $m_H = 1$, $m_O = 16$, $m_N = 14$ (in atomic mass units, *amu*) we have
$m_{fuel} = 2$ *amu*, $m_{air} = 0.5(32+3.76\times 28) = 68.64$ *amu*, so $f_{stoich} = 2.91\times 10^{-2}$
$\Rightarrow f = 0.85\, f_{stoich} = 2.48\times 10^{-2}$.
Alternatively one could work directly with the reaction written for $\phi = 0.85$.