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ASE 376K TEST #1 March 4, 2008

(Closed book section)

For multiple choice questions place an **x** in the appropriate column. Ambiguous selections will be treated as errors.

| 1. Flow in a non-ideal aircraft inlet: | Always rises | Always falls | Remains constant |
|---|---|---|---|
| Static pressure, $p$ | x__ | ___ | ___ |
| Stagnation pressure, $p_o$ | ___ | __x_ | ___ |
| Static temperature, $T$ | x___ | ___ | ___ |
| Stagnation temperature, $T_o$ | ___ | ___ | __x_ |
| Static density, $\rho$ | x___ | ___ | ___ |
| Entropy, $s$ | x___ | ___ | ___ |

(*3 points*)

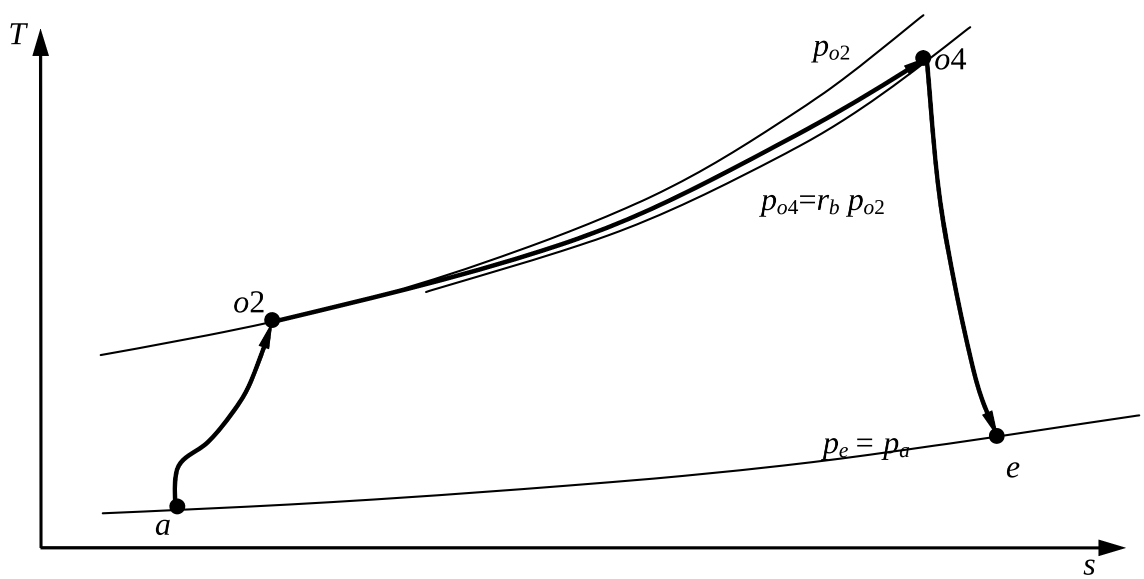
2. Explain why a converging nozzle is required to accelerate a subsonic flow, but a diverging nozzle is required to accelerate a supersonic flow. (*2 points*)

For any quasi-1D flow, $\rho A u$ = const, $dA/A = -d\rho/\rho - du/u$. When the flow is accelerated ($du>0$), the density decreases as $d\rho/\rho = -M^2\, du/u$ so $dA/A = (M^2-1)\, du/u$. In a subsonic flow $M < 1$, the decrease in density is more than offset by the increase in velocity, so the area must decrease to satisfy mass conservation. In a supersonic flow ($M>1$) the density decrease is so large that the area must increase to pass the flow and satisfy mass conservation.

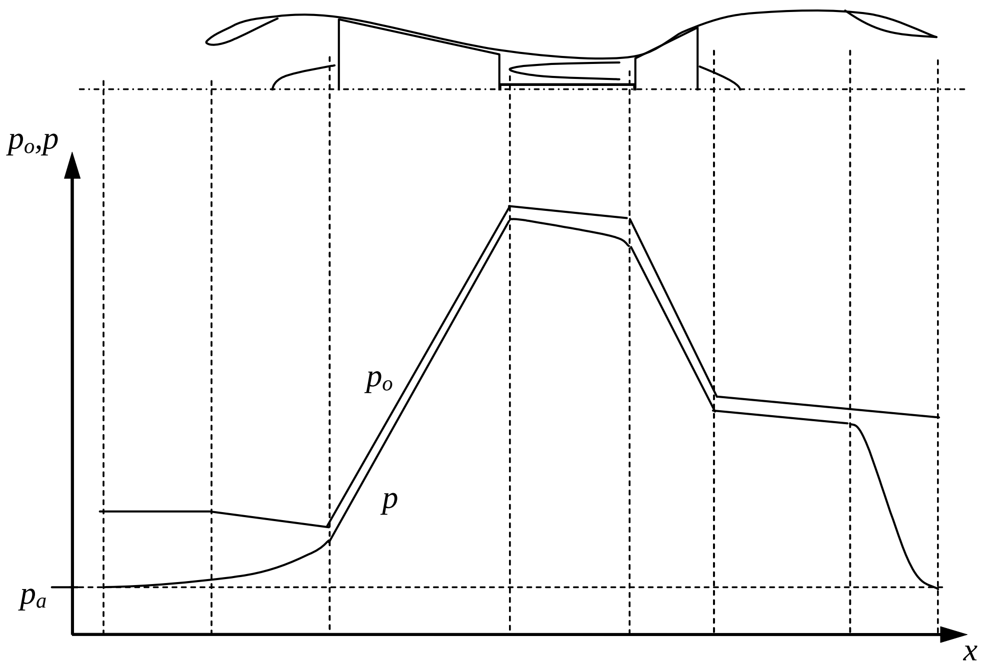
3. Define the propulsive efficiency of a turbojet engine. (*2 points*)

$$\eta_p = \frac{Thrust\ power}{Kinetic\ power\ added\ to\ fluid\ stream} = \frac{\Im \times u}{\dot{m}_a\left[(1+f)\frac{u_e^2}{2} - \frac{u^2}{2}\right]} = \frac{\left[(1+f)u_e - u\right]u}{\left[(1+f)\frac{u_e^2}{2} - \frac{u^2}{2}\right]}$$

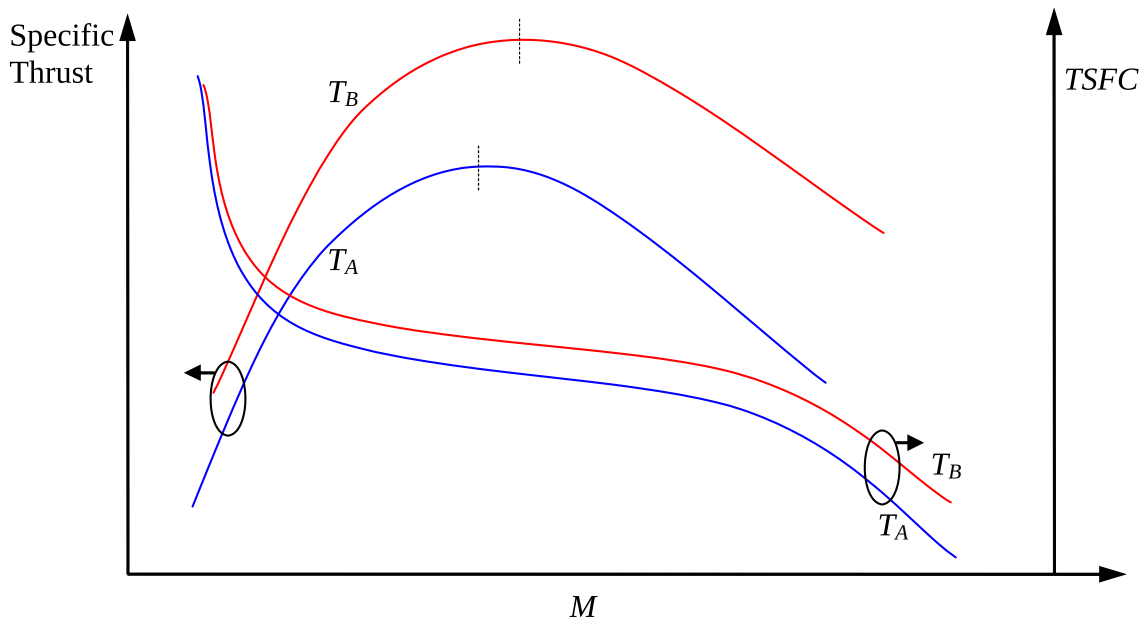
4. Sketch the T-s diagram for a non-ideal ramjet identifying the states $a$ (ambient), $o2$ (inlet to combustor), $o4$ (exit from combustor), $e$ (nozzle exit). Draw appropriate constant pressure lines and assume $p_e=p_a$. (*4 points*)



5. Sketch the variation of stagnation ($p_o$) and static ($p$) pressure for a *non-ideal* turbojet under sub-sonic cruise conditions ($M$~$0.9$) without afterburner. (*4 points*)

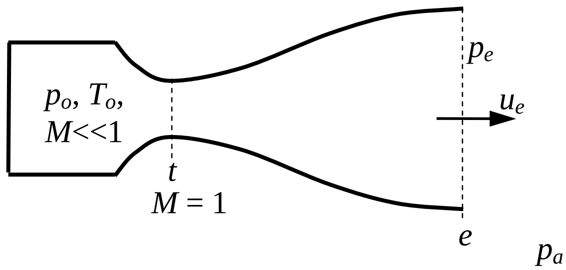



6. Sketch the variation of design Specific Thrust for a ramjet cycle as a function of *Mach* number at fixed engine maximum temperature $T_A$. Sketch a second curve for a different maximum temperature $T_B > T_A$. Similarly, sketch the variation of design Thrust Specific Fuel Consumption (*TSFC*) with $M$ for $T_A$ and $T_B$. Identify each curve clearly (variable plotted and temperature). (*4 points*)



**Need:** (a) $\Im$; (b) $A_e$; (c) $A_t$ for a rocket nozzle

**Given:** $\dot{m}$ = 50 *kg/s*, $\hat{M}_{prop}$ = 25 *kg/kmol*,
$\gamma$ = 1.28, $p_o$ = 2.40 *MPa*, $T_o$ = 2600 *K*,
$p_a$ = 101 *kPa*, $\eta_n$ = 0.95




**Solution:**

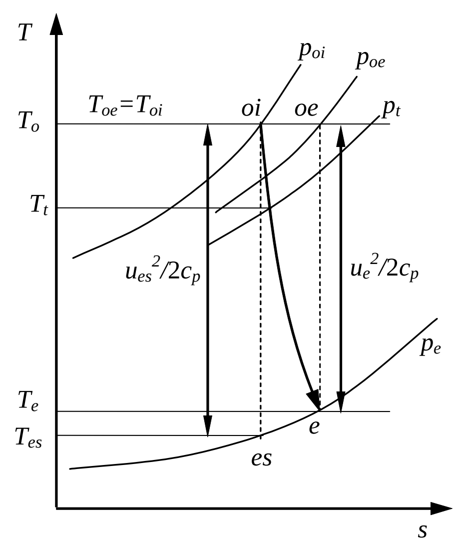
**Assumptions:**
(1) Steady, quasi-1D flow
(2) $p_e = p_a$ at design point
(3) Adiabatic nozzle
(4) Ideal gas behavior, with constant $c_p$
(5) Constant polytropic efficiency

$$\Im \overset{(1)}{=} \dot{m}u_e + \cancel{(p_e - p_a)}_{0(2)} A_e$$

$$T_{oe} \overset{(3)}{=} T_{oi} = T_e + \frac{u_e^2}{2c_p} = 2600\ K; \quad c_p \overset{(4)}{=} \frac{\gamma}{\gamma - 1} R_{prop};$$

$$R_{prop} \overset{(4)}{=} \frac{\hat{R}}{\hat{M}_{prop}} = \frac{8314\ J/kmol-K}{25\ kg/kmol} = 333\ J/kg\text{-}K.$$

So $c_p$ = 1520 *J/kg-K*




$$T_{es} = T_{oi}\left(\frac{p_e}{p_{oi}}\right)^{\frac{(\gamma-1)}{\gamma}} = 1300\ K$$

Then $u_e = \sqrt{2c_p\eta_n(T_o - T_{es})} = 1.94\times10^3\ m/s$;

$\Im = \dot{m}u_e = 9.69\times10^4\ N$; $\boxed{\Im = 97\ kN}$

$$\dot{m} \overset{(1)}{=} \rho_e A_e u_e \qquad \rho_e \overset{(4)}{=} \frac{p_e}{R_{prop}T_e} \overset{(2)}{=} \frac{p_a}{R_{prop}T_e}; \; T_e = T_o - \frac{u_e^2}{2c_p} = 1365\ K$$

$\rho_e$ = 0.222 *kg/m*$^3$

$$\Rightarrow A_e = \frac{\dot{m}}{\rho_e u_e} = \frac{50 kg/s}{0.222 kg/m^3 \times 1.94\times10^3 m/s}; \; \boxed{A_e = 0.116\ m^2}$$

To find $A_t = \dfrac{\dot{m}}{\rho_t u_t}$; $u_t = \cancel{M_t}_1\sqrt{\gamma R T_t}$;

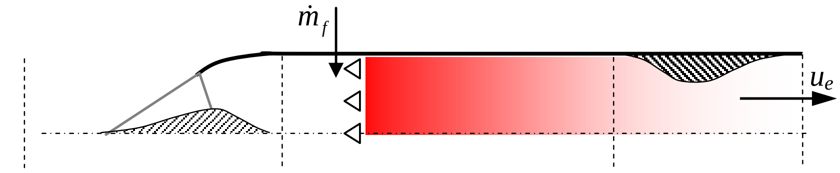
$$T_t = \frac{T_o}{1 + \frac{\gamma-1}{2}\cancel{M_t^2}_1} = \frac{2}{\gamma+1}T_o = 2281\ K; \; u_t = 985\ m/s; \; \rho_t \overset{(4)}{=} \frac{p_t}{R_{prop}T_t}$$

, need $p_t$. Assumption (5) is a reasonable approximation. Then $\dfrac{p_t}{p_o} \overset{(5)}{=} \left(\dfrac{T_t}{T_o}\right)^{\frac{\gamma}{\eta_{pe}(\gamma-1)}} = \left(\dfrac{T_t}{T_o}\right)^{\mu}$. Can find $\eta_{pe}$ (or equivalently $\mu$) from inlet and exit conditions: $\dfrac{p_o}{p_e} = \left(\dfrac{T_o}{T_e}\right)^{\mu} \Rightarrow \mu = \dfrac{\ln(p_o/p_e)}{\ln(T_o/T_e)}$ = 4.918 (Correspondingly $\eta_{pe}$ = 0.930).

Then $p_t = p_o\left(\dfrac{T_t}{T_o}\right)^{\mu}$ = 1.26 *MPa*; $\rho_t \overset{(4)}{=} \dfrac{p_t}{R_{prop}T_t}$ = 1.66 *kg/m*$^3$; $\boxed{A_t = \dfrac{\dot{m}}{\rho_t u_t} = 3.05\times10^{-2}\ m^2}$

**Need:** Specific thrust, $\dot{m}_f$ for a ramjet

**Given:** $M_a = 3.8$ $T_a = 216.7\ K$, $\Im = 50\ kN$, $|\Delta H_R| = 42\ MJ/kg$, $T_{max} = 2200\ K$, $\eta_d = 0.90$, $r_b = 0.92$, $r_n = 0.96$, $\eta_b = 0.98$, $\gamma_h = 1.32$




**Solution:**

By assumption (1) $\Im = \dot{m}_a\left[(1+f)u_e - u\right] + \cancel{(p_e - p_a)}_{0(3)} A_e$

$u = M_a\sqrt{\gamma R T_a}$ =1121 *m/s*

$T_{o2} = T_{oa} = T_a\left(1+\frac{\gamma_c - 1}{2}M_a^2\right)$ = 843 *K*, $T_{o4} = T_{max}$ = 2200 *K*

$c_{ph} = \frac{\gamma_h}{\gamma_h - 1}R_{air}$ = 1184 *J/kg-K*

$f = \frac{T_{o4} - T_{o2}}{\left[\frac{\eta_b |\Delta H_R|}{c_{ph}} - T_{o4}\right]}$ = $4.17\times10^{-2}$ (< 0.067 as required)

$\frac{p_{o6}}{p_e} = \frac{p_{o6}}{p_{o4}}\frac{p_{o4}}{p_{o2}}\frac{p_{o2}}{p_a}\cancel{\frac{p_a}{p_e}}_{1(3)} = r_n r_b\left(1+\eta_d\frac{\gamma-1}{2}M_a^2\right)^{\frac{\gamma}{\gamma-1}}$ = 78.1

By assumption (4): $T_{o6} = T_{o4}$ = 2200 *K*; $T_e = T_{o6}\left(\frac{p_e}{p_{o6}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}$ = 765 *K*

$u_e = \sqrt{2c_{ph}\left(T_{o6} - T_e\right)}$ = 1843 *m/s*

$\Im/\dot{m}_a = \left[(1+f)u_e - u\right] \Rightarrow$ $\boxed{\Im/\dot{m}_a = 799\ m/s}$

$\dot{m}_a = \frac{\Im}{(\Im/\dot{m}_a)}$ = 62.6 *kg/s* $\Rightarrow$ $\boxed{\dot{m}_f = f\dot{m}_a = 2.61\ kg/s}$

**Assumptions**

(1) Steady, quasi-1D flow
(2) Ideal gas behavior, $R_{air}$ = 287 J/kgK, piecewise constant $\gamma_c$, $\gamma_h$
(3) $p_e = p_a$ at design point
(4) Adiabatic components