S.1 All descriptions of a gas which can be treated analytically or computationally are the result of simplifying assumptions which are valid as limiting cases and break down when conditions exceed certain (somewhat fuzzy) limits.

Descriptions which specify properties at a point in a flow assume that these properties are meaningfully defined. This requires that there be a sufficient number of molecules within a small volume around that point so that statistical fluctuations are negligible. These arise because of changes in the exact number of molecules within that volume and their precise properties: identity, velocity, kinetic energy, etc. Assume that at least 1000 molecules must be sampled before statistical fluctuations are negligible. Then the volume needed to sample 1000 molecules must be small compared to the scale of gradients in the flow if properties are to be defined at a point. If L is a characteristic dimension associated with gradients in the flow field and δ is the mean distance between molecules, derive a relation between L and δ that must be satisfied if statistical fluctuations are to be negligible.

As pointed out in class, the conventional transport coefficients are not useful when the scale of gradients becomes smaller than the mean free path between collisions. The ratio (λ/L) is the Knudsen number, and the Navier-Stokes equations (which use these transport coefficients) become inadequate when $Kn > 0.1$ (say).

The assumption of a dilute gas explicitly assumes that the molecules spend most of their time traveling freely through space and only occasionally encounter other molecules, and these collisions are instantaneous. It is also implicitly assumed that only binary collisions occur, i.e. the probability of three molecules colliding simultaneously is negligibly small. This implies a limit on the ratio δ/d. Choose a limit that you feel is reasonable.

(a) Plot these three limits on the same (log-log) plot using dimensionless axes δ/d (x-axis) and L/d (y-axis) with δ/d in the range 1 - 1000.

(b) How are the Navier-Stokes equations to be interpreted in the domain where the continuum description is valid but significant statistical fluctuations exist?

S.2 Consider elastic collisions between molecules of two species A and B, with collision cross-section independent of relative speed, g. What fraction of these AB collisions have kinetic energy of relative motion, $\frac{1}{2}m_{AB}^{*}g^{2}$, less than $\frac{3}{2}kT$? What fraction of the collisions have kinetic energy after the collision less than $\frac{3}{2}kT$?

S.3 A gas mixture consists of molecules A and B of masses m_A and m_B respectively. The velocities distributions for each are Maxwellian. What is the distribution function of relative speed of type A molecules for type B molecules traveling at speed Z? This is the conditional speed distribution function $\chi(g|Z)$. Note that the velocity of A molecules of class C_i relative to B molecules of class Z_i, $g_i \equiv C_i - Z_i$, and speed $g = |g_i|$.

It may be helpful to consider the following chain of logic: For a molecule B with velocity Z_i what is the number (fraction) of molecules A with relative <u>velocities</u> g_i ? From this compute the fraction of A molecules with <u>speeds</u> g by integrating over the appropriate region of velocity space. (This integration is made much simpler if the coordinate reference frame is chosen so that the polar axis lies along the direction of Z_i.)This step gives the distribution of relative speeds for fixed Z_i. Finally integrate over all possible directions of Z_i keeping Z fixed.

S.4 An equilibrium perfect gas of a single species is confined in a container. What fraction of the molecules striking a wall of the container have a *normal* component of momentum that is greater than $m\overline{C}$?

S.5 In the study of very high temperature gases (e.g. in plasma physics) temperatures are often expressed in units of electron volts (*eV*). This equivalence is possible because temperature is a measure of energy via the kinetic definition of temperature. The numerical conversion is established by the equation: $kT = E$. Recall that 1 *eV* is the energy gained by an electron when it falls through a potential difference of 1 *V*. The charge on the electron is 1.602×10^{-19} *C*.

(a) Calculate the temperature equivalent of 1 *eV*.

(b) At what temperature (expressed in Kelvins) is a 5 *keV* plasma? (It is estimated that to achieve ignition of thermonuclear fusion reactions the average plasma temperature must be about 6.5 *keV*.)

(c) Calculate the average velocity of electrons and deuterons in the 5 *keV* plasma. The masses of the electron and deuteron are 9.108×10^{-31} *kg* and 3.343×10^{-27} *kg* respectively.

(d) When the space shuttle is in low-earth orbit (200 *km*), it moves at about 8 *km/s* relative to a very rarefied gas. O atoms in the gas striking the surface are responsible for the phenomenon of “shuttle glow”. What is the kinetic energy (in *eV*) of an O atom striking the surface at a relative speed of 8 *km/s*? What is the corresponding “temperature” (in *K)*?

S.6 “Maxwellian” molecules have a total collision cross section that is given by A/g, where A is a constant. For a gas consisting of a single species of such molecules:

(a) Find the collision rate per molecule, θ.

(b) Using the mean free path result for viscosity obtain an expression for the coefficient of viscosity μ, and indicate how μ varies with temperature and pressure.

[James C. Maxwell used this model for velocity dependent cross-sections because of the simple form one gets for collision integrals. Maxwellian molecules are considered to be too “soft”. In contrast, hard-sphere molecules, which may be described by $\sigma = A/g^n$, $n\to\infty$, are too “hard”. For many molecules n = 9 - 11 seems to fit experimental data over modest temperature ranges, though the best value for exponent n is a function of the nominal temperature: n tends to increase with T.]